ABSTRACT

The study of automorphisms of algebraic structures has contributed immensely to many important findings in mathematics. For example, Galois characterized the general degree five single variable polynomials f over Q, by showing that the roots of such polynomials cannot be expressed in terms of radicals, through the automorphism groups of the splitting field of f. On the other hand, the symmetries of any algebraic structure are captured by their automorphism groups. The study of completely primary finite rings has shown their fundamental importance in the structure theory of finite rings with identity. It is well known that quite reasonable research has been done towards characterization of the unit groups, R^* of certain classes of finite commutative completely primary rings. Much less known however, is whether there is a complete description of $Aut(R^*)$, the automorphism groups of the unit groups of these classes of rings. The existing literature is still scanty on the characterization of the structures of the automorphism groups of R^*. Therefore, in this thesis, we have characterized the structures and orders of the automorphisms of the unit groups of three classes of commutative completely primary finite rings, namely; Square Radical Zero, Cube Radical Zero and Power Four Radical Zero finite commutative completely primary rings. The unit groups of these rings are expressible as $R^* \cong \not\subset_{p^r-1} \times (1+J)$ where (1 + J) is a normal subgroup of R^* and J is the Jacobson radical of R. We have made use of the invertible matrix approach, the properties of diagonal matrices and determinants to count the number of automorphisms of (1+*J*). We have then adjoined the counted $Aut(1+J)$ to $\varphi\left(\not\subset_{p^r-1}\right)$ where φ is the Euler's phi-function, in order to completely characterize the order $Aut(R^*)$. Moreover, we have made use of the First Isomorphism Theorem to establish the relationship between $\left|GL_{rk(1+J)}(F_p)\right|$ and $\left|SL_{rk(1+J)}(F_p)\right|$. We noticed that our automorphisms yielded very unique structure and order formulae, distinct from the well-known structures and order formulae of the automorphisms of the cyclic groups C_n. The results obtained in this thesis contribute significantly to the existing literature on the structure theory of finite rings with identity, thereby providing a much needed, accessible modern treatment and a complete characterization of these classes of rings up to isomorphism.