Abstract

The numerical range of an operator on a Hilbert space has been extensively researched on. The concept of numerical range of an operator goes back as early as 1918 when Toeplitz defined it as the field of values of a matrix for bounded linear operators on a Hilbert space. Major results like convexity, that is the Toeplitz-Hausdorff theorem, the relationship of the spectrum and the numerical range, the essential spectra and the essential numerical range, have given a lot of insights. Most of these results have been on Hilbert spaces. As for Banach spaces there is still work to be done. There is scanty literature on the properties of the essential spectra and the essential numerical ranges on Banach spaces. The objectives of this study were to determine the properties of the essential spectrum and the properties of the essential numerical range, and to investigate the relationship between the essential spectrum and the essential numerical range for operators on Banach spaces. To study the properties of the essential spectra, we defined various parts of the spectra and using known theorems, we established the duality properties of these parts. For the essential numerical range, we applied the approach of Barraa and M¨ uller which considers a measure of noncompactness instead of the usual essential norm on the Calkin algebra. We finally extended the existing relations between the spectra and the numerical range to the setting of the essential spectrum and essential numerical range. We hope that the results of this study will be significant to both Applied Mathematicians and theoretical physicists for further research.