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Models for Level Premiums Payable to Benevolent Funds

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Abstract

The application of multiple life actuarial calculations have been studied by many authours for instance Elizondo [3] studied the construction of multiple decrement models from associated single decrement experiences.He posits that it is convenient to use the survival functions for the projection of future obligations in cash flows. Bowers [2] studied the actuarial calculations which are common in estate and gift taxation.The actuarial calculation is also common in insurance where stipulated payment called the benefit, one party (the insurer) agrees to pay to the other (the policyholder or his designated beneficiary) a defined amount (the claim payment or benefit) upon the occurrence of a specific loss while the insured pays periodic payment called premium. SACCOs and institution provide benevolent in terms of insurance against some losses, especially death. Unfortunately such organizations determine their premiums arbitrarily, thus one cannot tell whether such products are degenerating or not,this is because in such bodies benevolent funds and the mainstream operation fund are usually confounded.In this paper we develop models for level premiums for Saccos and Institutions providing benevolent funds, that is premiums is independent on the number of beneficiaries.We will use models of joint life,last life and multiple decrements to develop this model.

Keywords: Contributor, Beneficiary, Actuarial Present Value, Annuity, Joint Life, Multiple Decrement and Premiums

Introduction

The application of multiple life actuarial calculations are common in estate and gift taxation, for example, the investment income from a trust can be paid to a group of heirs as long as at least one of the group survives that is up to the last death , the principal from such trust is usually donated to the institutions or charitable organizations.[2].

Another model which is common is multiple decrement ,example of multiple decrement is where an employee can withdraw, become disabled or retires, each of this are called decrements. Multiple decrements model is used to predict, the net probabilities of decrements and survivals as well as the force of decrements which are major factors in determining the magnitude of decrements.The principle of multiple decrements is applied in the construction of life tables which enables prediction of the expected time and age an individual will attain before termination in a cohort[1]

We determine the models for premium which are independent on the number of survivors of the beneficiaries to be paid by a policyholder to the benevolent fund.We look at different cases involving the contributor and various number of beneficiaries, this will enable us to develop generalized case of contributor and n beneficiaries. Next we look at different possible cases

Case one

Here we consider the case of contributor (x_1) and one beneficiary (x_2), for example contributor and spouse. The following scenarios are possible.

1. Both contributor (x_1) and the beneficiary (x_2) survives the term period n benefit paid will be zero with a

   probability of ${}_np_{x_1x_2}$, the Actuarial present value of the benefit will be;

$$\bar{A}_{x_1x_2:\overline{n}|} = 0 \times \int_0^n v^t {}_np_{x_1x_2} dt = 0 \tag{1}$$

2. Contributor (x_1) dies before the end of the term period and (x_2)survives (x_1), a benefit of $= b_1$ with probability ${}_np_{x_2\,t_1}q_{x_1}$

$$\bar{A}_{x_1^1x_2:\overline{n}|} = b_1 \int_0^n v^t {}_tp_{x_1x_2}\mu_{x_1+t}(t)dt \tag{2}$$

3. Beneficiary (x_2) dies before the term period and (x_1) survives (x_2). Benefit of b_2 for the event of death of x_2 and a benefit of b_1 for the event of

death of x_1 with probability ${}_nq_{x_1x_2^1} + {}_nq_{x_2n}p_{x_1}$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1x_2^1:\overline{n}|} = b_2\int_0^n v_{2t_2}^t p_{x_1\,t_2}p_{x_2}\mu_{x_2+t_2}(t)dt_2 + v^{t_2}{}_{t_2}p_{x_1x_2}b_1\int_{t_2}^n v^{t_1}{}_{t_1}p_{x_1+t_2}\mu_{x_1+t_2+t_1}(t_1)dt_1 \quad (3)$$

In the event that x_1 survives the term, the second part of the above equation will be zero, therefore the benefit paid will be b_2. Hence the equation reduces to

$$\bar{A}_{x_1x_2^1:\overline{n}|} = b_2\int_0^n v^t{}_{t_1}p_{x_1\,t_2}p_{x_2}\mu_{x_2+t_2}(t_2)dt_2$$

The Premium

The premium paid by the contributor x_1 is given by

$$P = \frac{\bar{A}_{x_1^1x_2:\overline{n}|} \times {}_np_{x_2\,t_1}q_{x_1} + \bar{A}_{x_1x_2^1:n} \times {}_nq_{x_1x_2^1} + {}_nq_{x_2n}p_{x_1}}{\bar{a}_{x_1x_2:n}}$$

That is, the premium is the sum of the actuarial present values multiplied by their probabilities divided by the annuity.

Case two

consider the contributor (x_1) and two beneficiaries $(x_2), (x_3)$

1. $(x_1), (x_2), (x_3)$ survives the term period, the benefit paid $= 0$ with probability of ${}_np_{x_1x_2x_3}$ Actuarial present value of the benefit will be;

$$A_{x_1x_2x_3:n} = 0 \times \int_0^n v^t{}_np_{x_1x_2}dt = 0 \quad (4)$$

2. The contributor (x_1) dies before the term period ends and beneficiaries x_2 and x_3 survives contributor (x_1) with probability of ${}_np_{x_2x_3}q_{x_1}$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1^1x_2x_3:\overline{n}|} = b_1\int_0^n v_{1t_1}^t p_{x_1x_2x_3}\mu_{x_1+t_1}(t_1)dt_1 \quad (5)$$

3. Beneficiary x_2 survives the contributor and beneficiary (x_3) and the contributor (x_1) dies before the term period ends and (x_1) survives (x_3) with a probability of ${}_nq_{x_1^2x_3^1x_2}$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1^2x_2x_3^1:\overline{n}|} = b_3\int_0^n v^{t_3}\,{}_{t_3}p_{x_1x_2x_3}\mu_{x_3+t_3}(t_3)dt_3 + b_1v^{t_3}\,{}_{t_3}p_{x_1x_2}\int_{t_3}^n v^{t_1}\,{}_{t_1}p_{x_1+t_3:x_2+t_3}\mu_{x_1+t_3+t_1}(t_1)dt_1 \tag{6}$$

4. Beneficiary x_3 survives the contributor but (x_2) dies and the contributor (x_1) survives beneficiary (x_2) with probability of ${}_np_{x_3}q_{x_2^1x_1^1}$ with benefit of $b_2 + b_1$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1^2x_2^1x_3:\overline{n}|} = b_2\int_0^n v^{t_2}\,{}_{t_2}p_{x_1x_2x_3}\mu_{x_2+t_2}(t_2)dt_2 + b_1v^{t_2}\,{}_{t_2}p_{x_1x_3}\int_{t_2}^n v^{t_1}\,{}_{t_1}p_{x_1+t_2:x_3+t_2}\mu_{x_1+t_2+t_1}(t_1)dt_1 \tag{7}$$

5. Beneficiary (x_3) dies and beneficiary (x_2) survives beneficiary(x_3), then (x_1) survives (x_2) with probability of ${}_nq_{x_1x_2^2x_3^1}$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1^3x_2^2x_3^1:\overline{n}|} = b_3\int_0^n v^{t_3}\,{}_{t_3}p_{x_1x_2x_3}\mu_{x_3+t_3}(t_3)dt_3 + b_2v^{t_3}\,{}_{t_3}p_{x_1x_2}\int_{t_3}^n v^{t_2}\,{}_{t_3}p_{x_1+t_3:x_2+t_3}\mu_{x_2+t_3+t_2}(t_2)dt_2 \tag{8}$$

$$+ b_1v^{t_2+t_3}\,{}_{t_2+t_3}p_{x_1}\int_{t_2}^n v^{t_1}\,{}_{t_1}p_{x_1+t_2+t_3}\mu_{x_1+t_2+t_3+t_1}dt_1$$

Another possible scenario from the above equation are;

(a) Beneficiary (x_2) dies before the term period and the rest survives the term period,equation (4.5) reduces to

$$= b_3\int_0^n v^{t_3}\,{}_{t_3}p_{x_1x_2x_3}\mu_{x_3+t_3}(t_3)dt_3$$

(b) Beneficiary (x_3) dies before the term period and the rest survives the term period,equation (4.5) reduces to

$$= b_2\int_0^n v^{t_2}\,{}_{t_2}p_{x_1x_2x_3}\mu_{x_2+t_2}(t_2)dt_2$$

(c) Beneficiaries (x_3) and (x_2) dies before the term period and the rest survives the term period, equation (4.5) reduces to

$$= b_3 \int_0^n v^{t_3} {}_{t_3}p_{x_1x_2x_3}\mu_{x_3+t_3}(t_3)dt_3 + b_2 v^{t_3} {}_{t_3}p_{x_1x_2} \int_{t_3}^n v^{t_2} {}_{t_3}p_{x_1+t_3:x_2+t_3}\mu_{x_2+t_3+t_2}(t_2)dt_2$$

6. The contributor (x_1) and Beneficiaries $(x_2),(x_3)$ dies before the term ends, but, (x_2) dies, then (x_3) survives (x_2) and (x_1) survives (x_3) with probability of ${}_nq_{x_2^1 x_3^2 x_1^3}$ the benefits paid is $b_2 + b_3 + b_1$ Actuarial present value of the benefit will be;

$$\bar{A}_{x_1^3 x_2^1 x_3^2:\overline{n}\rceil} = b_2 \int_0^n v^{t_2} {}_{t_2}p_{x_1x_2x_3}\mu_{x_2+t_2}(t_2)dt_2 + b_3 v^{t_2} {}_{t_2}p_{x_1x_2} \int_{t_2}^n v^{t_3} {}_{t_2}p_{x_1+t_2:x_3+t_2}\mu_{x_3+t_2+t_3}(t_3)dt_3 \tag{9}$$

$$+ b_1 v^{t_2+t_3} {}_{t_2+t_3}p_{x_1} \int_{t_2}^n v^{t_1} {}_{t_1}p_{x_1+t_2+t_3}\mu_{x_1+t_2+t_3}dt_1$$

The Premium

$$P = \frac{\bar{A}_{x_1x_2x_3:\overline{n}\rceil} \times {}_np_{x_1x_2x_3} + \bar{A}_{x_1^1 x_2x_3:\overline{n}\rceil} \times {}_np_{x_2x_3}q_{x_1} + \bar{A}_{x_1^2 x_2 x_3^1:\overline{n}\rceil} \times {}_nq_{x_1^2 x_3^1 x_2} + \bar{A}_{x_1^2 x_2^1 x_3:\overline{n}\rceil} \times {}_np_{x_3}q_{x_2^1 x_1^1}}{\bar{a}_{x_1x_2x_3:\overline{n}\rceil}} +$$

$$\frac{\bar{A}_{x_1^3 x_2^2 x_3^1:\overline{n}\rceil} \times {}_nq_{x_1 x_2^2 x_3^1} + \bar{A}_{x_1^3 x_2^1 x_3^2:\overline{n}\rceil} \times {}_nq_{x_2^1 x_3^2 x_1^3}}{\bar{a}_{x_1x_2x_3:\overline{n}\rceil}}$$

The premium is obtained by adding all actuarial present value divided by the annuity paid by the contributor.

Generally

Generally for $n-1$ beneficiaries $(x_2),(x_3),(x_4),\therefore,(x_n)$ and the contributor

1. When the beneficiaries survives the contributor (x_1)

$$\bar{A}_{x_1^1 x_2x_3x_4\ldots x_n:\overline{n}\rceil} = b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1x_2\therefore x_n}\mu_{x_1}(t_1)dt_1$$

2. When the contributor survives beneficiary (x_2) and the remaining beneficiaries survives (x_1)

$$\bar{A}_{x_1^2 x_2^1 x_3 x_4 \ldots x_n:\overline{n}|} = b_2 \int_0^n v^{t_2} {}_{t_1}p_{x_1 x_2 \therefore x_n} \mu_{x_2}(t_2) dt_2 + b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1 x_2 \therefore x_n} \mu_{x_1}(t_1) dt_1$$

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The contributor (x_1) survives beneficiary (x_j)

$$\bar{A}_{x_1^{j-1} x_2^1 x_3 x_4 \ldots x_n^j:\overline{n}|} = b_j \int_0^n v^{t_j} {}_{t_j}p_{xx \therefore x_{j-1} x_{j+1} \therefore} + \therefore + b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1 x_2 \therefore x_n} \mu_{x_1}(t_1) dt_1$$

Therefore the premium is the sum of the actuarial present value times their probability divided by the annuity paid by the contributor.

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