ANALYSIS OF HEAT AND MASS TRANSFER EFFECTS ON STEADY BUOYANCY DRIVEN MAGNETOHYDRODYNAMICS FLUID FLOW PAST AN INCLINED INFINITE FLAT PLATE

BY

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Abstract

The phenomenon of natural convection arises in fluids when temperature change causes density variation leading to buoyancy forces acting on the fluid particles. Natural convection flows are frequently encountered in physical and engineering problems such as chemical catalytic reactors, nuclear waste materials, solar collectors, thermal regulation process, security of energy systems etc. When a conductive fluid moves through a magnetic field and an ionized gas is electrically conductive, the fluid may be influenced by the magnetic field. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow. Such oscillatory flow has applications in industrial and aerospace engineering. However, the temperature and concentration do not remain constant in so many fluid flow problems of practical interests. Moreover, in natural convection flows, thermal input occurs at a surface that is itself curved or inclined with respect to the direction of gravity field. As a result, we considered a plate at varying, linear temperature distribution to approach such real cases. In pursuit of the objectives of the study, the effects of heat and mass transfer on a two dimensional boundary layer of a steady free convection magnetohydrodynamics (MHD) fluid flow on an inclined heated plate in which the angle of inclination is varied has been studied. The fluid was taken as viscous, incompressible, electrically conducting over a heated inclined flat plate. The plate wall and the ambient fluid medium were maintained at constant and different levels of temperature and concentrations such that the heat and mass transfer occurs from plate wall to the fluid medium. Also, due to coupling between the fluid velocity field and thermal/concentration fields, different complex behaviours were expected. To control such processes, we investigated the problem of combined heat and mass transfer permeated by a uniform transverse magnetic field in MHD free convection adjacent to an inclined surface by taking into account effects of viscous dissipation with sinusoidally varying surface temperature on velocity, temperature and concentration. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. The mathematical formulation yielded a set of governing partial differential equations (PDEs)

under a set of appropriate boundary conditions. The PDEs were transformed into ordinary differential equations (ODEs) by some similarity transformation, which were then solved using the shooting iteration technique with the fourth order Runge-Kutta numerical method together with the Secant technique of root finding to determine their solutions. Computations were performed for a wide range of the governing flow parameters and the effects of these flow parameters on the velocity, temperature and concentration shown graphically. From the graphical analysis, it was established that the flow field and other quantities of physical interest are significantly influenced by these parameters. The results of this study of flow over inclined surface is utilized as the basis of many scientific and engineering applications, as the technique of inclination which enhances cooling of materials is significant in industrial processes as cooling of towers, nuclear reactor cooling and metallurgical processes. Finally, employment of an external magnetic field has predominant role in material manufacturing industries as a control mechanism due to generation of Lorentz force.

CHAPTER ONE

INTRODUCTION

1.1 Background of the study

A fluid is a substance whose constituent particles may continuously change their positions relative to one another when shear force is applied to it. As fluid flows, heat is transferred from one point to another. This form of heat transfer in fluids is known as convection. Note that fluids do not exist in isolation rather in solid containers, as such, fluids flowing in engineering devices occur within magnetic field. Fluid flow in the presence of a magnetic field is called hydromagnetic flow, and the study of hydromagnetic flows is called Magnetohydrodynamics (MHD).

Magnetohydrodynamics (MHD) is therefore, is the fluid mechanics of electrically conducting fluids in the presence of magnetic field, which is either applied externally or self-generated within the fluid by inductive action. Some of these fluids include liquid metals (such as mercury, molten iron) and ionised gases known by physicists as plasma, an example being the solar atmosphere. The official origin of MHD dates back to pioneering discoveries of Hartmann and Lazarus [21], where they conducted a theoretical and experimental investigation of MHD flows in ducts using mercury and observed that a force is produced on the fluid in the direction normal to both applied electric and magnetic fields. Thereafter, a Swedish physicist Hannes Alfven [3] in the context of plasma physics applications studied magnetohydrodynamics of astrophysical phenomenon as independent scientific discipline. Later, several authors [4, 32, 35, 45, 52] extended the research on MHD in regard to its immense applications, precisely, in many engineering applications, the knowledge of MHD is very vital as it has been used to explain certain phenomena in the universe [12]. This has led to intensive scientific research in the field of computational modelling of MHD fluid flow.

Convective fluid motion is defined as the collective motion of particles of a fluid. In this regard, there are two types of convection flow, forced convection and natural or free convection. Forced convection occurs when an external driving force causes the fluid to flow. For instance, use of a fan, a pump or a blower. Situations for which there is no forced velocity, yet convection currents exist within the fluid are referred to as free or natural convection. Free convection flow occurs frequently in nature and in such flows, the velocity and temperature distributions are coupled. The flow originates due to buoyancy force which is induced by temperature difference between the surface and the fluid. Such flows (free convection) find relevance in many applications, for instance, it strongly influences the operating temperatures of power generating and electronic devices thereby plays a major role in a vast array of thermal manufacturing applications. Secondly, it's equally important in establishing temperature distributions within buildings and in determining heat losses or heat loads for heating, ventilating, and air conditioning systems. Further, free convection distributes the poisonous products of combustion during fires. To environmental sciences, it is relevant as well since it drives oceanic and atmospheric motions, and the related heat and mass transfers processes. It is therefore justifiable that free convection is a buoyant flow that develops due to fluid density gradients and a body force whose combined effect induces free convection current. Such buoyancy driven flows are termed natural convection (NC) flows and can be modelled mathematically [6, 27, 33, 34, 42]. Notice that in such buoyancy induced flows, heat is transferred as a result of temperature gradients. It is definite that the energy transfer is always from the higher temperature medium to the lower temperature one. The science that deals with the determination of such energy transfer is heat transfer. Hence, the study of heat transfer forms an integral part of natural convection flow and basically belongs to the class of problems in boundary layer theory. The quantity of heat transferred is highly dependent upon the fluid motion within the boundary layer. Many practical heat transfer applications involve the conversion of some form of mechanical, electrical, nuclear or chemical energy to thermal energy in the medium.

Along with the free convection flow, the phenomenon of mass transfer is equally very common since temperature and concentration are coupled. When a system contains two or more components whose concentration vary from point to point, then there is a natural tendency for mass to be transferred, minimizing the concentration differences within the system. Convection mass transfer therefore, involves the transport of materials between boundary surface and moving fluid. Mass transport always plays an important role in many industrial processes, for instance, removal of pollutants from plant discharge. An interest to investigate this phenomenon and in particular, the case of mass transfer on free convectional flow, is noble. Several researchers have studied problem of free convection flow [19, 11, 28, 48, 55]. However, in the mentioned investigations, viscous dissipation is neglected yet such effects are important in geophysical flows. For instance, in the design of MHD generators and accelerators, in certain industrial operations as in polymer processing flows such as injection moulding or extrusion at high rates. Since viscous dissipation changes the temperature distribution by playing a role like an energy source, it leads to affected heat transfer rates and thus are usually characterised by Eckert number (Ec).

Also, during convective motion, both heat and mass of fluid are transferred by a phenomenon called double diffusion or combined heat and mass concentration transfer convection. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies as the earth and so on.

In many natural and technological processes, temperature and mass or concentration diffusion act together to create a buoyancy force which drives the fluid. Because of the coupling between the fluid velocity field and the diffusive (thermal and concentration) fields, double-diffusive convection is more complex than the convective flow which is associated with a single diffusive scalar, and many different behaviours may be expected. Such double-diffusive processes have applications in many of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering among others. Such coupled heat and mass transfer convection flows occur frequently in nature not only due to temperature difference, but also as a result of concentration differences or rather a combination of these two. In addition, such processes are only of interest in the boundary layer, however, the boundary layer flows adjacent to inclined plates or wedges have received little attention. Recall that there is no component of the buoyancy force along the surface, therefore the accelerating flow must be driven indirectly by a buoyancy induced pressure gradient which leads to the variation in wall temperature. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, resulting in a variation in the flow. This finds application in nuclear engineering, where cooling of medium is more important from safety point of view. During this cooling process the plate temperature starts oscillating about a non-zero constant mean temperature, hence the justification of sinusoidally varying surface temperature. Such, oscillatory flow has applications in industrial and aerospace engineering. Numerous authors [45, 38, 28, 26], have studied MHD free convection flow with some extended effects along vertical or horizontal plates. However, MHD free convection flow with some extended effects along an inclined plate with sinusoidal temperature conditions has received inadequate attention yet in many natural convection flows, the thermal input occurs at a surface that is itself curved or inclined with respect to the direction of the gravity field. Equally, it is noticed that temperature and concentration do not remain constant in so many fluid flow problems of practical interest, thereby a need to consider a plate at varying angles to approach such real cases. Further, partial differential equations exhibit different general solutions depending on imposed boundary conditions hence we anticipate a unique solution for this choice of boundary conditions. Therefore, the study considered heat and mass transfer characteristics phenomenon on MHD free convection steady buoyancy induced flow of an incompressible, electrically conducting fluid over an inclined heated infinite plate with varied inclination angle under the influence of an applied uniform magnetic field. In addition, we also considered combined effect of double diffusion, where dissipation and thermal diffusion taken into account with periodically varying surface temperature, when the temperature of the plate oscillates periodically about a constant mean temperature.

1.2 Statement of the problem

Convection is a major mode of heat and mass transfer in fluids and plays an important role in a wide range of fields such as engineering, science and industry. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow, which has applications in nuclear engineering, where cooling of medium is more important from safety point of view. However, the temperature and concentration do not remain constant in so many fluid flow problems of practical interests. Moreover, in natural convection flows, thermal input occurs at a surface that is itself curved or inclined. Further, due to coupling, double-diffusive convection is characterised by complexity than one associated with a single diffusive scalar, and many different complex behaviours could be expected. In this thesis therefore, we considered an inclined heated plate at varying angles of inclinations under the influence of applied uniform magnetic field. An analysis of effects of heat and mass transfer permeated by a uniform transverse magnetic field in MHD free convection adjacent to an inclined surface, taking into account viscous dissipation with sinusoidally varying surface temperature on velocity, temperature and concentration was done to control such processes .

1.3 Objectives of the study

General objective

The general objective of this study was to investigate the effects of heat and mass transfer on a steady buoyancy driven MHD fluid flow past an inclined infinite flat plate under Boussinesq model.

Specific objectives

The key objectives of this study were;

(i) To formulate a mathematical model describing heat and mass transfer effects on steady buoyancy driven MHD fluid flow past an inclined infinite flat plate.

(ii) To numerically solve the formulated problem in (i), with periodic surface boundary conditions..

(iii) To analyze the effect of variation of inclination angle of the plate so as to determine the optimal inclination for effective fluid flow.

(iv) To analyze the effects of parameter variation on velocity, temperature and concentration profiles.

1.4 Significance of the study

This study of natural convection flows finds application in material manufacturing industries as a control mechanism where there arise situations in which the heat generated during certain operations may be disadvantageous to the production of equipment. The undesirable amount of heat can be removed as much as possible by Lorentz force, that is generated by the existence of magnetic field. Secondly, such flows likewise find relevance in many applications, for instance, it strongly influences the operating temperatures of power generating and electronic devices thereby plays a major role in a vast array of thermal manufacturing applications, in establishing temperature distributions within buildings and in determining heat losses or heat loads for heating, ventilating, and air conditioning systems.

Moreover, buoyancy induced convective flow is key in many heat removal processes in engineering technology since both science and technology have interest in passive energy storage systems, such as the cooling of spent fuel rods in nuclear power applications and the design of solar collectors.

Last but not least, the study of flow past an inclined surface can be utilized as the basis of many scientific and engineering applications, including earth science, nuclear engineering and metallurgy. In nuclear engineering, it finds its applications for the design of the blanket of liquid metal around a thermonuclear fusion-fission hybrid reactor. In metallurgy, it can be applied during the solidification process. The results of the problem are also of great interest in geophysics, in the study of interaction of geomagnetic field with the fluid in the geothermal region.

1.5 Research Methodology

The study of natural convection buoyancy induced, heat and mass transfer effects in the presence of a magnetic field past an inclined surface are mostly experienced in industries and engineering which are practical in nature. To formulate this problem, consider the heat and mass transfer of a steady two-dimensional laminar flow of a viscous, incompressible, electrically conducting and dissipating fluid moving past an inclined infinite plate. The motion is in the presence of a uniform magnetic field of intensity B_0 applied normal to the plate surface. Assume the x axis of a cartesian coordinate system (x, y) is directed along the plate and the y axis is perpendicular to the plate surface. Then the origin of the coordinate system is taken to be the leading edge of the plate. The acceleration due to gravity g is taken to be acting vertically downwards. The plate surface is inclined to the vertical direction by an angle γ. This formulation yields a set of non linear coupled partial differential equations (PDEs) governing the motion of fluid under a set of appropriate boundary conditions [47].

To achieve the second objective, the system of non linear coupled PDEs so formulated were reduced by suitable similarity techniques, mainly because similarity technique transforms easily the PDEs into a set of non linear coupled ordinary differential equations (ODEs). Such ODEs were solved numerically under the boussinesq approximation, using shooting method with the fourth order Runge-Kutta method and the Secant technique of root finding.

In the analysis, a Mathematica program was developed in order to analyze the effects of variation of inclination angle γ and pertinent parameters involved. The results obtained were displayed in graphs.

1.6 Preliminary Concepts

This section gives detailed information on concepts used in the thesis. However, these concepts are not exhaustive, therefore, most of these literature can be found in [13, 23, 24, 25, 37, 47, 48, 51, 54, 60], for further reading.

1.6.1 Laminar and Turbulent flows

1.6.1.1 Laminar flow

A fluid is a substance which offers no resistance to shear deformation and the continuous deformation of a substance is known as flow. A smooth regular flow in layers is called laminar flow. Here, fluid particles remain in motion in respective layers or no fluid particles will be exchanged from one layer to another. As a result, laminar flow is also referred to as streamline or viscous flow. These terms are descriptive of the flow because in laminar flow;

(i) Fluid particles move in definite and observable paths or streamlines

(ii) The flow is characteristic of viscous (thick) fluid or is one in which viscosity of the fluid plays a significant part.

1.6.1.2 Turbulent flow

When the motion becomes disorderly, eddies and vortices occur, then it is known as turbulent motion. The pattern of flow in this case is no longer smooth and stable but becomes irregular and chaotic. It is complex flow and the pattern changes continuously with time. The velocity of the particles at each given point varies abruptly with time. A transition from laminar flow to turbulent flow occurs very suddenly as the flow rate increases, therefore, fast flow increases the chance of turbulence. When the flow becomes turbulent there is a decrease in the volume flow rate. Therefore, when a fluid flows around an object the shape of the object is a very important parameter in determining the type of flow. For instance, the flow of blood through a normal artery is laminar. However, when irregularities occur in artery the flow becomes turbulent.

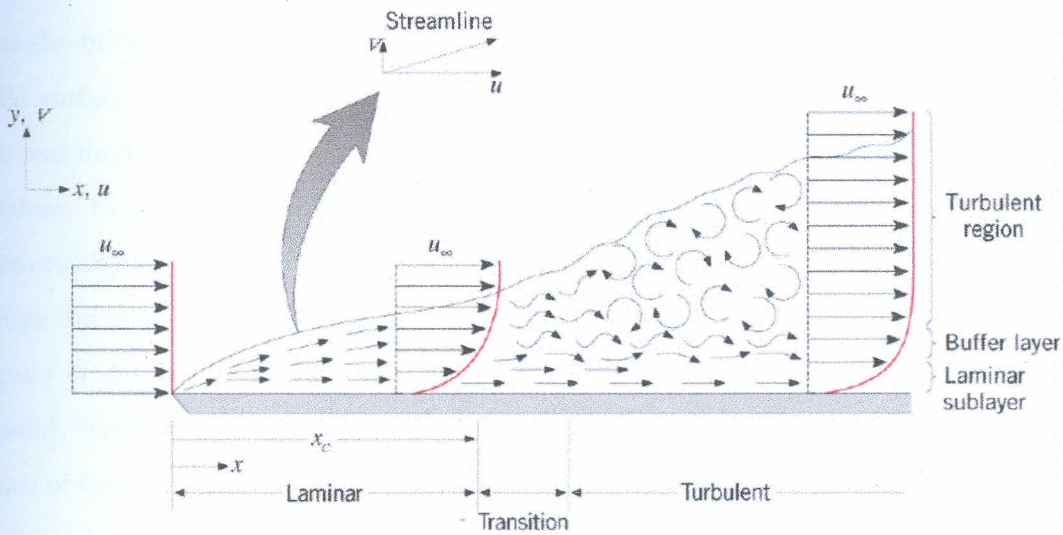

Figure 1.6.1: Boundary Layer Transition (Source: [54])

1.6.2 Classification of Flow Phenomena

1.6.2.1 Steady and unsteady state flows

Steady-state flow refers to the condition where the fluid properties at any single point in the system do not change over time t while the flow in which any one of these parameters changes with time is considered to be unsteady. These fluid properties include temperature, density, velocity among others. One of the most significant properties that is constant in a steady-state flow system is the system mass flow rate, i.e. there is no accumulation of mass within any component in the system.

1.6.3 Viscosity and Incompressibility

1.6.3.1 Viscosity

Viscous fluids tend to be gooey or sticky, indicating that fluid parcels do not slide past one another, or past solid surfaces very readily (but in a fluid they do always slide). Such fluid interacts with the surface it comes in contact with resulting in a kind of fluid friction called viscosity. It is thus a property of a fluid which determines its resistance to shearing stresses between the layers of the fluid i.e. it measures the resistance of the fluid to deforming due to a shear force.

It can also be thought of as the internal friction of a fluid which makes it resist flowing past a solid surface or other layers of the fluid. This property of fluid is especially important right near fluid boundaries because viscous forces bring the fluid to a complete stop at the boundary. In addition, it stabilizes those flows that would otherwise become turbulent, or chaotically unsteady. The boundary effect produces drag forces on objects moving through fluids and therefore, viscosity depends on fluid properties and dimensions. The viscosity of the fluid in motion cannot be neglected in all regions, since at the surface of the solid, there is a significant condition known as the no-slip condition i.e. flow at the surface of the body is at rest relative to that body. At a certain distance from the body, the viscosity of the flow can again be neglected since it is usually significantly dependent on the temperature of the fluid and relatively independent of the pressure. For most fluids, as the temperature of the fluid increases, the viscosity of the fluid decreases. An example of this can be seen in the lubricating oil of engines. When the engine and its lubricating oil are cold, the oil is very viscous or thick, after the engine is started and the lubricating oil increases in temperature, the viscosity of the oil decreases significantly and the oil seems much thinner. In mathematics, this can be expressed as;

$$\tau = \mu \frac{\partial u}{\partial y}$$

where τ is called the shear stress and μ is the coefficient of dynamic viscosity. Such fluids which obey this relation are referred to as Newtonian fluids.

1.6.3.2 Incompressibility

Fluids are either gases or liquids that take the shape of the container. In fluid dynamics, the compressibility of a fluid is a very important factor. In nature, all the fluids are compressible. The concepts of compressible and incompressible fluids play a major role in fields such as fluid dynamics, fluid statics, aviation and many other fields. It is vital to have a proper understanding in the concepts of compressibility of fluids in order to understand such fields. An incompressible fluid is a fluid that does not change the volume due to external pressure. Most of the basic calculations done in fluid dynamics are done assuming the fluid is incompressible. The approximation of incompressibility is acceptable for most of the liquids as their compressibility is very low. However, the compressibility of gases is high, so gases cannot be approximated as incompressible fluids. The compressibility of an incompressible fluid is always zero (do not change in volume) whereas , compressible fluids reduce in volume when an external pressure is applied.

1.6.4 Natural Convection and Buoyancy Induced Flows

The convective heat transfer depends also on the details of the flow field, this leads to further classification of convection based on the flow field namely, natural or free convection and forced convection.

1.6.4.1 Natural Convection

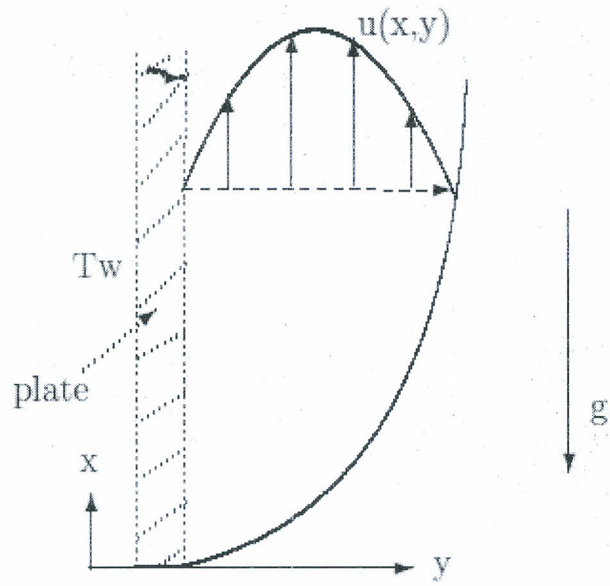

Figure 1.6.2: Convection flow (Source: [43])

In free or natural convection, the macroscopic fluid motion is due to body forces and their dependence on fluid density, which itself is sensitive to the temperature or the concentration (or both) of the species that constitute the fluid. Free convection is common in nature and has numerous applications and occurrences in industry. It is a major cause for atmospheric and oceanic recirculation and plays an increasingly important role in the passive emergency cooling systems of advanced nuclear reactors. During heating process of a fluid, the fluid density varies with temperature, and a flow is induced due to the force of gravity acting on the density variations. The buoyancy results in a force acting on the fluid, and the fluid would accelerate continuously if it were not for the existence of viscous forces. Therefore, viscous forces oppose the buoyancy forces thereby causing the fluid to move with a velocity distribution which creates a balance between the opposing buoyancy and viscous forces. Thus, buoyancy driven flows are classified as viscous flows of incompressible fluids. As such, free convection can be categorised as a buoyant flow that develops due to fluid density gradients and a body force whose combined effect induces free convection current (results from body force due to gravitational field acting on a fluid with density gradient ($\Delta\rho$ due to ΔT)). An interesting property about natural convection is that the velocity and temperature fields are completely coupled which makes any analytical analysis extremely difficult. Furthermore, the velocities encountered in natural convection flows are relatively low, therefore, the momentum and viscous effects are of the same order.

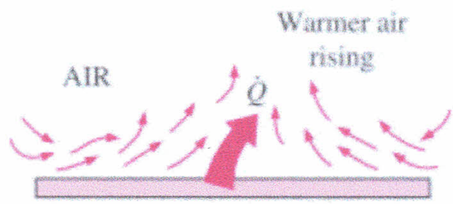

Figure 1.6.3: Free convection (Source: [54])

1.6.4.2 Forced Convention

In this type of flow, fluid arises from an external agent, for instance, a fan, a blower, the wind, or the motion of the heated object itself, which imparts the pressure to drive the flow.

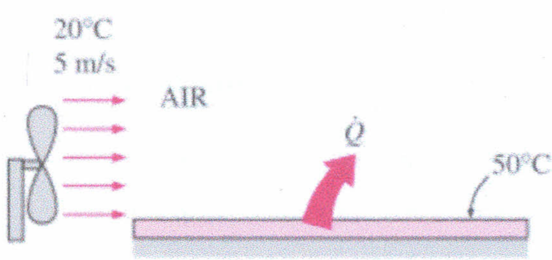

Figure 1.6.4: Forced convection (Source: [54])

1.6.4.3 The Boussinesq Model

The governing equations for natural convection flow are coupled partial differential equations and are therefore of considerable complexity. Another problem in obtaining a solution to these equations lies in the inevitable variation of the density ρ with temperature or concentration. Several approximations are generally made to simplify these equations. The most important one being the Boussinesq approximations. The Boussinesq approximation involve two aspects first, the density variation in the continuity equation is neglected. Second, the density difference, which causes the flow, is approximated as a pure temperature or concentration effect (i.e., the effect of pressure on the density is neglected). Therefore, for many natural convection flows, the Boussinesq model provides a faster convergence than just setting up the problem with fluid density as a function of temperature (ρ varies with respect to T). In fact, the density difference is estimated for thermal buoyancy as

$$(\rho - \rho_0)g \approx -\rho_0\beta(T - T_0)g \tag{1.6.1}$$

where ρ_0 - is the constant density of the flow at temperature T_0, which is the operating temperature. Equation 1.6.1 is obtained by using the Boussinesq approximation $\rho = \rho_0(1 - \beta\Delta T)$ to eliminate ρ from the buoyancy term. This approximation is accurate as long as changes in actual density are small, an important condition for the validity of these approximations is that

$$\beta(T - T_0) \leq 1$$

[25]. Therefore, the approximations are valid for small temperature differences if β is essentially unchanged.

1.6.5 Convection Heat Transfer

Convection heat transfer takes place between a surface and a moving fluid, when they are at different temperatures. In a strict sense, convection is not a basic mode of heat transfer as the heat transfer from the surface to the fluid consists of two mechanisms operating simultaneously. The first one is energy transfer due to molecular motion (conduction) through a fluid layer adjacent to the surface, which remains stationary with respect to the solid surface due to no-slip condition. Superimposed upon this conductive mode is energy transfer by the macroscopic motion of fluid particles by virtue of an external force, which could be generated by a pump or fan (forced convection) or generated due to buoyancy, caused by density gradients.

When a fluid flows over a surface, its velocity and temperature adjacent to the surface are same as that of the surface due to the no-slip condition. The velocity U_∞ and temperature T_∞ far away from the surface may remain unaffected. The region in which the velocity and temperature vary from that of the surface to that of the free stream are called hydrodynamic and thermal boundary layers, respectively. The velocity tends to vary from zero (when the surface is stationary) to its free stream value U_∞. This happens in a narrow region where there is a sharp velocity gradient. The narrow region is called hydrodynamic boundary layer. In the hydrodynamic boundary layer region the inertial terms are of same order of magnitude as the viscous terms. Similar to the velocity gradient, there is a sharp temperature gradient in this vicinity of the surface if the temperature of the surface of the plate is different from that of the flow stream. This region is called thermal boundary layer, denoted as δt, as displayed in the diagram below;

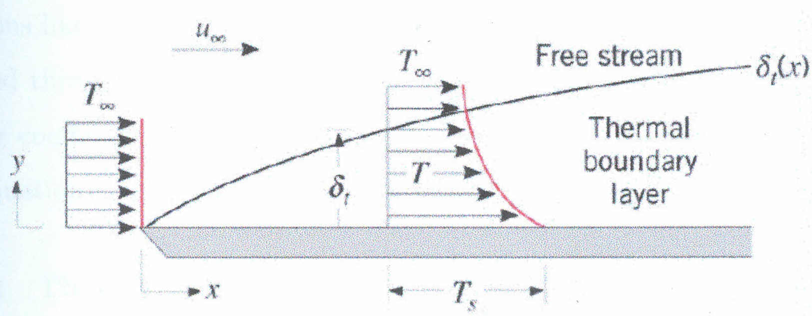

Figure 1.6.5: Thermal boundary layer (Source: [54])

In the thermal boundary layer region, the conduction terms are of same order of magnitude as the convection terms. The momentum transfer is related to kinematic viscosity ν while the diffusion of heat is related to thermal diffusivity α hence the ratio of thermal boundary layer to viscous boundary layer is related to the ratio $\frac{\nu}{\alpha}$, which is Prandtl number. Therefore, the ratio of thermal boundary layer thickness to the viscous boundary layer thickness depends upon Prandtl number. The rate of heat transfer Q due to free convection is described by Newtons law of cooling

$$Q = hA(T_w - T_\infty) \quad or \quad hA\triangle T$$

where h is the convection heat transfer coefficient, A is the surface area of plate, T_w is the temperature of the plate wall, T_∞ is the temperature of the surrounding and $\triangle T$ is the temperature difference. The temperature gradient near the wall depends on the rate at which the fluid near the wall can transport energy into the mainstream. Thus the temperature gradient depends on the flow field, with higher velocities able to pressure sharper temperature gradients and hence higher heat transfer rates. Hence, determination of convection heat transfer requires the application of laws of fluid mechanics in addition to the laws of heat transfer.

1.6.5.1 Determination of convective heat transfer coefficient

Evaluation of convective heat transfer coefficient is difficult as the physical phenomenon is quite complex. Analytically, it can be determined by solving the mass, momentum and energy equations. However, analytical solutions are available only for very simple situations like $1-$ Dimensional cases, hence most of the convection heat transfer data is obtained through careful experiments, and the equations suggested for convective heat transfer coefficients are mostly empirical. Since the equations are of empirical nature, each equation is applicable to specific cases.

1.6.5.2 Thermal Diffusivity

The product ρC_p, which is frequently encountered in heat transfer analysis is called the heat capacity of a material. Both the specific heat C_p and the heat capacity ρC_p, represent the heat storage capability of a material. Another very important material property that appears in the transient heat conduction analysis is the thermal diffusivity which represents how fast heat diffuses through a material and is defined as

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p}$$

Note:

The larger the thermal diffusivity, the faster the propagation of heat into the medium while the smaller this value means that heat is absorbed by the material and thus a small amount of heat will be conducted further.

1.6.6 Fundamentals of mass transfer

When a system contains two or more components whose concentration vary from point to point, there is a natural tendency for mass to be transferred, minimizing the concentration differences within the system. The transport of one constituent from a region of higher concentration to that of lower concentration is called **mass transfer**. A common example of mass transfer is drying of a wet surface exposed to unsaturated air. Refrigeration and air conditioning deal with processes that involve mass transfer. Some basic laws of mass transfer relevant to refrigeration and air conditioning are discussed below.

1.6.6.1 Fick's Law of Diffusion

This law deals with transfer of mass within a medium due to difference in concentration between various parts of it. This is very similar to Fouriers law of heat conduction as the mass transport is also by molecular diffusion processes. According to this law, rate of diffusion of component $A(kg/s)$ is proportional to the concentration gradient and the area of mass transfer, i.e.

$$m_A = -D_{AB}A\frac{dC_A}{dx}$$

where, C_A is concentration of the surface, D_{AB} is called diffusion coefficient for component A through component B, and it has the units of m^2/s just like those of thermal diffusivity α and the kinematic viscosity of fluid ν for momentum transfer.

1.6.6.2 Convective mass transfer

Mass transfer due to convection involves transfer of mass between a moving fluid and a surface or between two relatively immiscible moving fluids. Similar to convective heat transfer, this mode of mass transfer depends on the transport properties as well as the dynamic characteristics of the flow field. Similar to Newton's law for convective heat transfer, the convective mass transfer equation can be written as:

$$m = h_m A \bigtriangleup C_A$$

where h_m is the convective mass transfer coefficient and ΔC_A is the difference between the boundary surface concentration and the average concentration of fluid stream of the diffusing species A.

Similar to convective heat transfer, convective mass transfer coefficient depends on the type of flow i.e. laminar or turbulent and forced or free. In general, the mass transfer coefficient is a function of the system geometry, fluid and flow properties and the concentration difference. Similar to momentum and heat transfers, concentration boundary layers develop whenever mass transfer takes place between a surface and a fluid.

1.6.7 Natural convection due to combined thermal and mass diffusion effects

Mass transfer in natural convection is more complicated than in forced convection. The reason is that non uniformity in the chemical species concentrations, which is usually the main cause of diffusive mass transfer, also contributes to non uniformity in the fluid mixture density. The non uniformity in the fluid density will then contribute to the buoyancy driven flow. Thus, unlike forced convection in which the effect of diffusive mass transfer on the hydrodynamics is often negligible, mass diffusion can have a significant effect on the overall phenomenology of buoyancy-driven flows. Buoyancy driven flows caused by non uniformity of humidity in air and in buildings, and caused by non uniformity of salinity in seawater, are some examples.

When heat and mass transfer are both present, we then deal with buoyancy driven flows caused by combined thermal and mass diffusion. It is important to note that, unlike forced convection, the analogy between heat and mass transfer cannot be applied to derive correlations for mass transfer based on the modification of correlations for natural convection heat transfer. The analogy based methods for obtaining mass transfer correlation by manipulating heat transfer correlations (and vice versa) can be applied under only very restrictive, limiting conditions.

1.6.8 Boundary layer theory and equations

A region of fluid in the immediate vicinity of a bounding surface where strong gradients of velocity (and potentially, other variables such as temperature) occur is known as boundary layer. Although the layer is thin, it is very important to know the details of flow within it. The main flow velocity within this layer tends to zero while approaching the wall (no-slip condition) to a maximum value. This thickness is usually expressed by

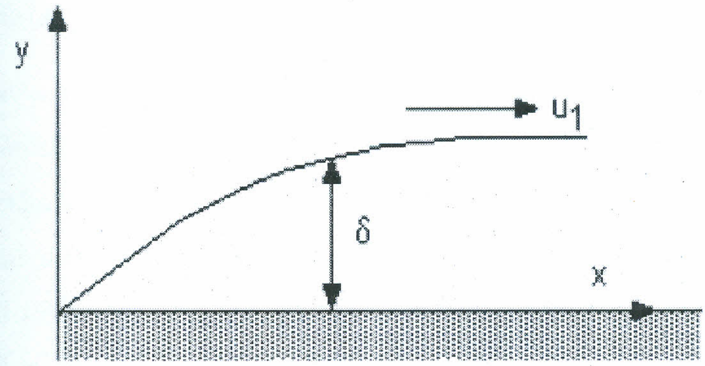

Figure 1.6.6: Boundary layer (Source: [1])

the symbol δ. The impact of the no-slip boundary condition at the surface of the object will extend only through this thin layer of fluid, and beyond it the fluid acts essentially as an inviscid fluid. In other words, outside the boundary layer the flow field does not feel the viscous effect caused by the presence of the object. It feels only the blockage caused by the presence of the object, as a result of which the streamlines in the flow field become curved around the object. Also the gradient of this velocity component in a direction normal to the surface is large as compared to the gradient in the stream wise direction. This can also be seen as the layer of fluid in which the tangential component of the velocity of the fluid relative to the body increases from zero at the surface to the free stream value at some distance from the surface.

The velocity boundary layer results from the difference between the free stream velocity and the zero velocity at the wall, while the thermal boundary layer results from a difference between the free stream and surface temperatures. Note that since the fluid is considered to be a binary mixture of species A and B, the concentration boundary layer originates from a difference between the free stream and surface concentrations.

Therefore, the boundary layer concept is a very important tool and allows for the simplification of the analysis of virtually all transport processes in two important ways.

(i) First, it limits the domain in the flow field where the viscous and other effects of the wall must be included in the conservation equations

(ii) Second, it shows that, within the boundary layer, the conservation equations can be simplified by eliminating certain terms in those equations.

1.6.8.1 Boundary layer on a flat plate

Air flowing past a solid surface will stick to that surface. This phenomenon caused by viscosity is a description of the no-slip condition. This condition states that the velocity of the fluid at the solid surface equals the velocity of that surface. The result of this condition is that a boundary layer is formed in which the relative velocity varies from zero at the wall to the value of the relative velocity at some distance from the wall.

Consider the flow of a fluid parallel to a thin, flat plate, as shown in figure (1.6.7). Away from the wall the fluid has a uniform velocity profile. This is the simplest physical condition as far as the phenomenology of boundary layers is concerned and produces effects that with some variations apply to other configurations as well. For the thin plate depicted in figure (1.6.7), measurements slightly above and below the plate would agree with the predictions of the inviscid flow theory. Very close to the wall, however, a non-uniform velocity profile would be noted in which, over a very thin layer of fluid of thickness δ, the fluid velocity increases from zero (at $y = 0$) to U_∞ (at $y = \delta$). The velocity of the fluid actually approaches U_∞ asymptotically, and δ is often defined as the normal distance from the wall where $\frac{u}{U_\infty} = 0.99$ or $\frac{u}{U_\infty} = 0.999$.

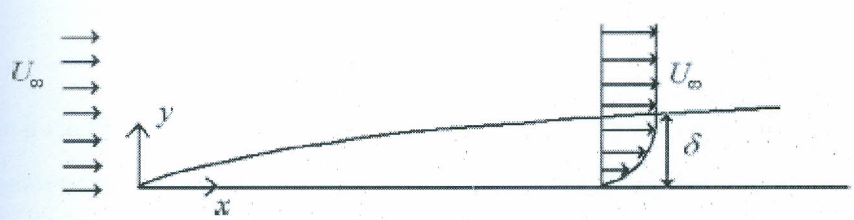

Figure 1.6.7: Laminar flow boundary layer on a flat plate (Source: [37])

1.6.8.2 Boundary layer equations for laminar flow

Motion of a fluid in which there are coexisting velocity, temperature, and concentration gradients must comply with several fundamental laws of nature. In particular, at each point in the fluid, conservation of mass, energy, and chemical species, as well as Newtons second law of motion, must be satisfied. Equations representing these requirements are derived by applying the laws to a differential control volume situated in the flow. We begin by restricting attention to applications for which body forces are negligible i.e.

free flow ($x = y = 0$), there is no thermal energy generation ($q = 0$), and the flow is non reacting. Additional simplifications may be made by invoking approximations pertinent to conditions in the velocity, thermal, and concentration boundary layers. The boundary layer thickness are typically very small relative to the size of the object upon which they form, and the x - direction velocity, temperature, and concentration must change from their surface to their free stream values over these very small distances. Therefore, gradients normal to the objects surface ($y-$ direction), are much larger than those along the surface ($x-$ direction). As a result, we can neglect terms that represent x - direction diffusion of momentum, thermal energy, and chemical species, relative to their y - direction counterparts as below

$$\frac{\partial^2 u}{\partial x^2} \leq \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial^2 T}{\partial x^2} \leq \frac{\partial^2 T}{\partial y^2}, \quad \frac{\partial^2 C}{\partial x^2} \leq \frac{\partial^2 C}{\partial y^2}. \tag{1.6.2}$$

By neglecting the x - direction terms, we are assuming that the corresponding shear stress, conduction flux, and species diffusion flux are negligible.

Furthermore, because the boundary layer is so thin, the x - direction pressure gradient within the boundary layer can be approximated as the free stream pressure gradient as

$$\frac{\partial P}{\partial x} \approx \frac{\partial P_\infty}{\partial x}. \tag{1.6.3}$$

The form of $P_\infty(x)$ depends on the surface geometry and may be obtained from a separate consideration of flow conditions in the free stream. Hence, the pressure gradient may be treated as a known quantity. With the foregoing simplifications and approximations, the overall continuity equation is unchanged

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1.6.4}$$

where u - is component of velocity in x - direction and v - is component of velocity in y- direction. Now if a fluid flows past a solid, a fluid layer is formed adjacent to the boundary of the solid. This layer is called a boundary layer and strong viscous effects exist within this layer. Assuming steady 2-D incompressible flow, then the x component of the momentum equation is given by

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - g + \nu\frac{\partial^2 u}{\partial y^2}, \tag{1.6.5}$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, $\frac{\partial P}{\partial x}$ is the free stream pressure gradient in the quiescent region outside the boundary layer. And in this region $u = 0$, therefore equation (1.6.5) reduces to

$$\frac{\partial P}{\partial x} = -\rho_\infty g.$$

The y-momentum equation results in $\frac{\partial P}{\partial y} = 0$, therefore $P = P(x)$, it then implies that

$$\frac{\partial P}{\partial x} = -\rho_\infty g.$$

Note that the pressure gradient inside the boundary layer must balance the pressure gradient outside the boundary layer i.e.,

$$\left(\frac{\partial P}{\partial x}\right)_{\text{in boundary layer}} = -\rho g \text{ (outside the boundary layer)}$$

so that equation (1.6.5) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}(-\rho_\infty g) - g + \nu\frac{\partial^2 u}{\partial y^2},$$

which implies that

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= g\left(\frac{\rho_\infty - \rho}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2} \\ &= g\left(\frac{\Delta\rho}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{1.6.6}$$

where $\Delta\rho = \rho_\infty - \rho$. The term $g\left(\frac{\Delta\rho}{\rho}\right)$ is the **buoyancy force**. If the density variations are due to temperature variations only, then the term may be related to a fluid property called volumetric thermal expansion coefficient.

1.6.8.3 Volumetric Thermal Expansion Coefficient

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p.$$

This thermodynamic property of the fluid provides a measure of the amount by which the density changes in response to a change in temperature at constant pressure (incompressible fluid). Thus,

$$\beta \approx -\frac{1}{\rho}\left(\frac{\Delta \rho}{\Delta T}\right) = -\frac{1}{\rho}\left(\frac{\rho_\infty - \rho}{T_\infty - T}\right) \quad \text{at constant pressure.}$$

Hence

$$(\rho_\infty - \rho) \approx \rho\beta(T - T_\infty) \tag{1.6.7}$$

This simplification is known as the **Boussinesq approximation**. Substituting equation (1.6.7) in equation (1.6.6), we have

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}.$$

Hence the relationship between the buoyancy force which drives the flow and the temperature is apparent. Since the buoyancy effects are confined in the momentum equation, the mass and energy conservation equations are unchanged.

The energy equation from control volume idea takes the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{1.6.8}$$

This equation results from application of conservation of energy to a differential control volume in the flowing fluid. Terms on the left-hand side account for the net rate at which thermal energy leaves the control volume due to bulk fluid motion (advection). The term on the right-hand side accounts for the net inflow of thermal energy due to y - direction conduction. The species conservation equation from same control volume analogy, is given by

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}. \tag{1.6.9}$$

where C_A is concentration of the surface, D_{AB} is called diffusion coefficient for component A through component B.

Similarly, this equation is obtained by applying conservation of chemical species to a differential control volume in the flow. Terms on the left-hand side account for net transport of species A due to bulk fluid motion (advection), while the right-hand side represents the net inflow due to y - direction diffusion.

Equations (1.6.4), (1.6.5), (1.6.8) and (1.6.9) may be solved to determine the spatial variations of u, v, T, and C_A in the different laminar boundary layers. For incompressible, constant property flow, equations (1.6.4) and (1.6.5) are uncoupled from equations (1.6.8) and (1.6.9). That is, equations (1.6.4) and (1.6.5) may be solved for the velocity fields, $u(x, y)$ and $v(x, y)$, without consideration of equations (1.6.8) and (1.6.9). therefore, velocity gradient $\left(\frac{\partial u}{\partial y}\right)|_{y=0}$ could then be evaluated, and the wall shear stress could be obtained from equation (1.6.5).

In contrast, through the appearance of u and v in equations (1.6.8) and (1.6.9), the temperature and species concentration are coupled to the velocity field. Hence $u(x, y)$ and $v(x, y)$ must be known before equations (1.6.8) and (1.6.9) may be solved for $T(x, y)$ and $C_A(x, y)$. Once $T(x, y)$ and $C_A(x, y)$ have been obtained from such solutions, the convection heat and mass transfer coefficients may then be computed.

1.6.8.4 Laminar boundary layer conservation equations

Consider the flow parallel to a flat plate (Figure 1.6.7). This is the simplest configuration, but provides information that is much more general. As a further simplification, let us assume constant properties and incompressible flow, without body force. Also, let us assume 2-dimensional (x, y) flow. Then the conservation equations for mass, momentum, energy and species become, respectively,

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1.6.10}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty). \tag{1.6.11}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{1.6.12}$$

1.6.9 Laminar Boundary Layer Thickness (Scale Analysis)

Scale analysis (or order of magnitude analysis) is very useful and widely used tool for solving problems in the area of heat transfer and fluid mechanics, pressure driven wall jet, separating flows behind backward facing steps, jet diffusion flames, study of linear and non-linear dynamics among others. Scale analysis is recommended as the premier method for obtaining the most information per unit of intellectual effort, despite the fact that it is a precondition for good analysis in dimensionless form. The object of scale analysis therefore is to use the basic principles of convective heat transfer to produce order of magnitude estimates for the quantities of interest. Hence, scale analysis anticipates within a factor of order one when done properly.

1.6.10 Order of magnitude analysis

The essential part of this argument is to recognize that boundary layers are (in general) thin in comparison to their length of development (except perhaps right at the start of the body). Hence $\frac{\delta}{L}$ is small, where δ is the thickness of the boundary layer and L is the length over which it develops. By order of magnitude we mean the size of the terms, which we will represent by $O(\cdot)$, i.e. this means is of the order of magnitude of. This sort of argument is often called a scaling argument. In essence what we are doing is looking at how the various terms in the equation change when we change the primary flow variables (such as the mean velocity, the size of the object we are studying, the viscosity of the fluid). To say that one variable scales with another quantity simply means that we expect it to increase proportionally when we increase that variable. This approach is very useful to a scientist or engineer since in this way we can determine which terms of an equation are likely to be important under certain conditions and then simplify the equation (by dropping those terms that are likely to be insignificant).

We consider an order of magnitude analysis of the two-dimensional conservation of mass equation for steady incompressible Newtonian laminar flow over a flat plate and simplify it using Prandtl ideas [51].

We shall use scaled variables, let L be the reference length, and U_∞ as a reference velocity. Then the symbols x^* and y^* are used for the scaled counterparts of the physical coordinates in the sketch (Figure 2.9.2), and the symbols u^* and v^* are used for the dimensionless counterparts of the physical velocity components in the x and y directions, respectively.

From the scaling, we know that u^* is $O(1)$. This means that the magnitude of u^* lies between 0 and a number that is of the order of unity. In this particular case, because the maximum value of the physical velocity is that of the uniform stream approaching the plate, namely U_∞, the maximum value of u is, in fact, precisely 1. But this is not necessarily the meaning implied by the order symbol that we are using. Note that the order of magnitude of a quantity is the same regardless of its sign.

Since the velocity u^* varies in the range mentioned above, while the scaled variable x^* also varies from 0 to 1 (we say $x \sim O(1)$), then a conclusion that the derivative $\frac{\partial u^*}{\partial x}$ is $O(1)$ as well is correct. The scaled incompressible version of the continuity equation is

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0$$

Then it is evident that $\frac{\partial u^*}{\partial x}$ and $\frac{\partial v^*}{\partial y}$ must sum to zero, this forces the derivative $\frac{\partial v}{\partial y}$ to be of $O(1)$ too.

We know that the variable $y \sim O(\delta)$ where δ represents the boundary layer thickness divided by the length L. In other words, δ is the scaled boundary layer thickness. Because the derivative $\frac{\partial v^*}{\partial y} \sim O(1)$, we must conclude that the change in the scaled velocity component v across the boundary layer must be of $O(\delta)$. Due to viscosity, we have the no slip condition at the plate i.e. $u = 0$ at $y = 0$. At infinity (outside the boundary layer), away from the plate we have that $u = U_\infty$ (constant speed) as $y \to \infty$. Note that δ is a very small quantity when the Reynolds number $Re_L \gg 1$ implying that $\delta \ll x$. Thus, the scaled velocity in the y direction in the boundary is a very small quantity.

1.6.11 Thermal boundary layers in laminar flow

The transfer of heat between a solid body and a liquid or gaseous flow is a problem whose consideration involves the science of fluid motion. In order to determine the temperature distribution it is necessary to combine the equations of motion with those of heat conduction. It is intuitively evident that the temperature distribution around a hot body in a fluid stream will often have the same character as the velocity distribution in boundary layer flow. For instance, if we imagine a solid body which is placed in a fluid stream and which is heated so that its temperature is maintained above that of the surroundings then it is clear that the temperature of the stream will increase only over a thin layer in the immediate neighbourhood of the body and over a narrow wake behind it.

The major part of the transition from the temperature of the hot body to that of the colder surroundings takes place in a thin layer in the neighbourhood of the body which, in analogy with flow phenomena, may be termed the thermal boundary layer. It is evident, that flow phenomena and thermal phenomena interact to a high degree. Hence it may be expected that in conjunction with the velocity boundary layer there will be formed a thermal boundary layer across which the temperature gradient is very large. It is therefore possible to take advantage of this fact and to introduce into the energy equation, which governs the temperature distribution, simplifications of a similar nature to those introduced earlier into the equations of fluid motion.

Consider a flat plate with a constant surface temperature. Assume a steady state, 2-dimensional flow field with constant properties and no viscous dissipation. The thermal energy equation and its boundary conditions for the boundary layer is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$T = T_w \quad at \quad y = 0$$

$$T = T_\infty \quad as \quad y \to \infty$$

We can recast these equations by using the dimensionless temperature:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

Then substituting in energy equation of the form

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\alpha\frac{\partial^2\theta}{\partial y^2}$$

with boundary conditions

$$\theta=1 \quad at \quad y=0$$

$$\theta=0 \quad as \quad y\rightarrow\infty$$

Note that temperature profiles for flow over an isothermal flat plate are similar to velocity profiles, hence, we expect a similarity solution for temperature to exist.

CHAPTER TWO

LITERATURE REVIEW

2.1 Related Literature

In literature, extensive research work has been performed to examine the effect of natural convection on flow past a surface.

Palani and Srikanth [45] presented an analysis to study the MHD flow of an electrically conducting, incompressible, viscous fluid past a semi-infinite vertical plate with mass transfer, under the action of transversely applied magnetic field. The dimensionless governing equations which were unsteady, two dimensional coupled and non-linear PDEs were solved using unconditionally stable and fast converging Implicit finite difference scheme.

The effects of rotation on the hydromagnetic free convection flow past an accelerated vertical plate with variable temperature and uniform mass diffusion was investigated by Muthucumaraswamy *et al.* [38]. The dimensionless governing equations were solved analytically using Laplace Transform Technique.

The unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous plate with suction taking into account the viscous dissipation was considered by Malga and Kishan [34], when the plate accelerates in its own plane, and the governing equations were solved numerically using Galerkin finite element method.

Muthucumaraswamy and Visalakshi [40], presented thermal radiation effects on unsteady free convection flow of a viscous incompressible flow past an exponentially accelerated vertical plate with variable temperature and uniform mass diffusion and the resulting governing equations were solved using Laplace Transform Method.

Unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux was studied by Ogulu and Makinde [41]. The boundary layer equations were derived and the resulting approximate nonlinear Ordinary Differential Equations (ODEs) were solved analytically using Asymptotic technique.

Mahmud and Sattar [30], researched on unsteady MHD free convection and mass transfer flow with Hall current, viscous dissipation and joule heating however, in this study thermal diffusion effects were neglected yet in convective fluid flow, when the flow is caused by temperature and concentration differences, thermal diffusion effect is key.

Viscous dissipation and mass transfer effects on unsteady MHD free convective flow along a moving vertical porous plate in the presence of internal heat generation and variable suction was investigated by Sharma *et al.* [53]. These studies concentrated on unsteady flows with vertical orientations and other physical extensions.

A numerical study of natural convection heat and mass transfer along a vertical wavy surface was performed by Jer-Huan *et al.* [26], where the wavy surface was maintained at uniform wall temperature and constant wall concentration. A simple coordinate transformation was employed to reduce the complex wavy surface to a flat plate and then solved using a marching finite difference scheme.

Muthucumaraswamy *et al.* [39], reported the exact solution to the problem of flow past an exponentially accelerated infinite vertical plate with variable temperature. The temperature of the plate was raised linearly with time t. The dimensionless governing equations were solved using Laplace Transform Technique.

Uwanta *et al.* [62], examined the effect of Viscoelastic fluid flow past an infinite plate with heat dissipation. Solutions to the dimensionless governing equations was done using perturbation technique.

Makinde [31] analyzed numerically the magnetohydrodynamics (MHD) boundary layer flow with heat and mass transfer over a moving vertical plate in the presence of magnetic field and a convective heat exchange at the surface with the surrounding. In his study, the governing boundary layer equations were transformed to a two point boundary value problem using similarity technique and the plate was considered at vertical orientation.

Mass transfer effects on steady two dimensional radiative MHD boundary layer flow over a non isothermal stretching horizontal sheet embedded in a porous medium was a study conducted by Vidyasagar *et al.* [20]. In their case, the governing system of PDEs were first converted to ODEs using similarity transformations and then finally solved by shooting method.

Abo-Eldahab and El-Aziz [14], performed an analysis on effect of Hall and ion-slip currents with internal heat generation or absorption on MHD free convection flow past a semi-infinite vertical flat plate. In their case, the governing differential equations were transformed by introduction of non- similarity variables and solved numerically by two point backward finite difference.

Palani and Abbas [44], investigated the combined effects of magnetohydrodynamics and radiation on free convection flow past an impulsively started isothermal vertical plate with Rosseland diffusion approximation. The fluid considered was a gray, radiation, absorbing, emitting but a non-scattering medium, with approximate transformations, the boundary layer governing the flow are reduced to non-dimensional equations valid in the free convection regime and were solved by the finite element method.

Kumar [29], examined on the combined effects of hall current, viscous dissipation, Joule heating and thermal diffusion on the hydromagnetic free convection and mass transfer flow of an electrically conducting, viscous incompressible fluid past an infinite vertical porous plate. The similarity solutions of the governing equations were solved analytically using perturbation technique with vertical orientation.

In the above mentioned studies, the plate has been considered in a vertical position with extended physical characteristics, rather than being inclined, however, in many natural convection flows, the thermal input occurs at a surface that is itself curved or inclined with respect to the direction of the gravity field.

Convection flows driven by temperature and concentration differences have been studied extensively in the past [7, 9, 59]. These previous studies of natural convection heat and mass transfer past a surface have focussed mainly on flat plate or regular ducts.

Ali *et al.* [4], concentrated on an exact analysis of combined effects of radiation and chemical reaction on the magnetohydrodynamics (MHD) free convection flow over an inclined plate embedded in a porous medium. The impulsively started plate with variable temperature and mass diffusion. The dimensionless momentum equation coupled with the energy and mass diffusion equations were analytically solved by Laplace Transform Method.

Finite difference analysis of natural convection flow over a heated plate with different inclination and stability analysis was considered by Begum *et al.* [6] and solved using Implicit finite difference method of Crank - Nicolson type.

Gnaneswara and Bhaskar [18], presented a numerical analysis on a steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface in the presence of heat generation and a porous medium. The governing partial differential equations were reduced to a system of ordinary differential equations by introducing similarity transformations which were solved by applying the Runge-Kutta method of fourth order with shooting technique.

Gebhart and Pera [16] studied the steady state natural convection on a vertical plate with variable surface temperature and variable mass diffusion. Using similarity techniques, they solved the boundary layer equations.

Sivasankaran *et al.* [58] analyzed Lie group analysis of natural convection heat transfer fluid flow past an inclined semi infinite surface in the presence of solute concentration. The governing partial differential equations were reduced to a system of ordinary differential equations by the translation and scaling symmetries.

Alam *et al* [2], studied the Hall effects on the steady MHD free convective flow and mass transfer over an inclined stretching sheet in the presence of a uniform magnetic field. The boundary layer equations were transformed by a similarity transformation into a system of coupled non linear ordinary differential equations and which were solved numerically by Runge- Kutta fourth-fifth order method using symbolic software.

Chen [10] performed an analysis to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration, taking into consideration the effects of ohmic heating and viscous dissipation. Power-law temperature and concentration variations are assumed at the inclined surface. The resulting governing equations were transformed using suitable transformations and then solved numerically by an implicit finite-difference method.

Ganesán and Palani [15], dealt with the unsteady natural convection past an inclined plate with variable heat and mass flux, under the influence of magnetic field.

The study of Sparrow *et al.* [60] is related to the convection flow about an inclined surface in which the combined force and free boundary layer problem has been discussed using the similarity method.

The effects of heat generation and thermophoresis on steady, laminar, hydromagnetic, two-dimensional flow with heat and mass transfer along a semi-infinite, permeable inclined flat surface was considered by Sattar *et al.* [50]. A similarity transformation was used to reduce the governing non-linear partial differential equations into ordinary ones which were then solved numerically by applying Nachtsheim-Swigert shooting iteration technique with sixth-order Runge-Kutta integration scheme.

Gnaneswara [17] employed scaling group of transformation for Heat and mass transfer effects on steady free convection flow in an inclined plate in the presence of MHD and viscous dissipation. The governing equations were reduced by similarity technique and solved using of Runge-Kutta fourth order along shooting method.

Details of effects of variable viscosity and thermal conductivity on MHD free convection and mass transfer flow over an inclined vertical surface in a porous medium with heat generation was investigated by Santana and Hazarika [49]. The flow governing equations were transformed to ordinary differential equations, which were numerically solved by Runge-Kutta method with shooting technique by using similarity transformation.

Manyonge *et al.* [36], examined the motion of a two dimensional steady flow of a viscous, electrically conducting, incompressible fluid flowing between two infinite parallel plates one of which is porous and under the influence of a transverse magnetic field and constant pressure gradient. The resulting coupled governing equation of motion were solved analytically and expression for the fluid velocity obtained was expressed in terms of Hartmann number.

Singh P. K [57], investigated on the effects of viscous dissipation on the MHD boundary layer flow adjacent to a an inclined plate in porous medium. The partial differential equations governing the boundary layer flow were converted into a system of ordinary differential equations by using suitable similarity transformations which were solved numerically.

The effects of viscous dissipation and joule heating on the flow of viscous incompressible fluid past a semi-infinite plate in the presence of a uniform transverse magnetic field was done by Hossain [22], but he did not consider thermal diffusion.

In many natural and technological processes, temperature and mass or concentration diffusion act together to create a buoyancy force which drives the fluid and this is known as double-diffusive convection, or combined heat and mass concentration transfer convection. Because of the coupling between the fluid velocity field and the diffusive (thermal and concentration) fields, double-diffusive convection is more complex than the convective flow which is associated with a single diffusive scalar, and many different behaviours may be expected. Such double-diffusive processes finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal among others.

CHAPTER THREE

FORMULATION AND SOLUTION OF THE PROBLEM

3.1 Formulation process

3.1.1 Scale Analysis

Let $u \sim U_\infty$ and $y \sim \delta$, since $\delta \ll x$, then by order of magnitude analysis, we have

$$\frac{\partial u}{\partial x} \approx \frac{U_\infty}{x}, \ \frac{\partial u}{\partial y} \approx \frac{U_\infty}{\delta}, \ \& \ \frac{\partial v}{\partial y} \approx \frac{v}{\delta}$$

substituting $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{U_\infty}{x} + \frac{v}{\delta} \approx 0$$

$$\Rightarrow v \sim \frac{U_\infty \delta}{x} \tag{3.1.1}$$

Similarly from the x- momentum equation given as,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{3.1.2}$$

we determine the second derivative $\frac{\partial^2 u}{\partial x^2}$, applying the idea of order of magnitude analysis we obtain

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{1}{x}\left\{ \left.\frac{\partial u}{\partial x}\right|_x - \left.\frac{\partial u}{\partial x}\right|_0 \right\} \approx \frac{U_\infty}{x^2}$$

Alternatively, since $\frac{\partial u}{\partial x} \approx \left(-\frac{U_\infty}{x}\right)$, substituting in the x - momentum equation (3.1.2), gives

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{U_\infty}{x}\right) \approx -U_\infty \frac{\partial}{\partial x}\left(\frac{1}{x}\right) \approx \frac{U_\infty}{x^2} \tag{3.1.3}$$

To estimate the order of magnitude of $\frac{\partial u}{\partial y}$, we first note that u varies from 0 to 1 across the boundary layer, while the variable y varies from 0 to δ. This is the reason for the estimate that $\frac{\partial u}{\partial y} \sim O(\frac{1}{\delta})$. To estimate the order of magnitude of the second derivatives, we must use similar arguments. In a like manner, the derivative $\frac{\partial u}{\partial y} \sim O(\frac{1}{\delta})$, which means that it varies from 0 to $\frac{1}{\delta}$ across the boundary layer, in a distance of the order δ. Therefore, the second derivative $\frac{\partial^2 u}{\partial y^2} \sim O(\frac{1}{\delta^2})$. i.e.

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{1}{\delta}\left\{ \left.\frac{\partial u}{\partial y}\right|_\delta - \left.\frac{\partial u}{\partial y}\right|_0 \right\} \approx \frac{1}{\delta}\left(0 - \frac{U_\infty}{\delta}\right) \approx \frac{U_\infty}{\delta^2} \tag{3.1.4}$$

Since the order of magnitude analysis provides useful information about the thickness of the boundary layer as well. Therefore, the order of magnitude of terms on the left and right hand sides of the x-momentum equation (3.1.2), becomes

$$U_\infty \frac{U_\infty}{x}, v\frac{U_\infty}{\delta} \approx \nu \frac{U_\infty}{\delta^2}. \tag{3.1.5}$$

Let $v \sim \frac{U_\infty \delta}{x}$, then equation (3.1.5) yields;

$$U_\infty \frac{U_\infty}{x}, \frac{U_\infty \delta}{x}\frac{U_\infty}{\delta} \approx \nu \frac{U_\infty}{\delta^2}$$

Since the orders of magnitude of terms are the same we have,

$$\delta^2 \sim \frac{\nu x}{U_\infty}$$

Implying that

$$\Rightarrow \delta \sim \sqrt{\frac{\nu x}{U_\infty}}. \tag{3.1.6}$$

Dividing through by x to express the result in dimensionless form gives

$$\frac{\delta}{x} \sim \sqrt{\frac{\nu}{U_\infty x}} = \frac{1}{\sqrt{Re_x}}.$$

Furthermore, because boundary layer approximations are valid only when $\frac{\delta}{x} \ll 1$, it is evident that such approximations make sense only $Re_x \gg 1$.

3.1.2 Similarity variable

In view of the fact that the velocity profiles at different locations along the plate are expected to be similar, let us use η as the independent variable, where this significant variable is expressed as $(\frac{y}{\delta})$ and we assume that the velocity may be expressed as a function of this variable, to obtain $\frac{u}{U_\infty} = g\left(\frac{y}{\delta}\right) = g(\eta)$. It follows that

$$\eta \sim \frac{y}{\delta} = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} = y\sqrt{\frac{U_\infty}{\nu x}}. \tag{3.1.7}$$

η is known as the similarity variable and $g(\eta)$ is the function we seek as a solution.

3.1.3 Variable Transformation

In order to solve a similarity problem, it is convinient to introduce an auxiliary function $\psi(x, y)$, called the stream function, which is useful when rewriting the two-dimensional Navier-Stokes equations. It is defined as

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}.$$

With the velocity field expressed in ψ, the continuity equations is automatically satisfied which is easily shown by inserting the above equations of u and v into the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

Thus

$$\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial^2\psi}{\partial x\partial y} = 0$$

Now, considering $u = \frac{\partial\psi}{\partial y}$, we have $d\psi = udy$ and from $\frac{u}{U_\infty} = g(\eta)$, we can express ψ as

$$\begin{aligned} \psi &= \int U_\infty g(\eta)dy \\ &= \int U_\infty\sqrt{\frac{\nu x}{U_\infty}}g(\eta)d\eta \end{aligned}$$

where $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ and $dy = \frac{d\eta}{\sqrt{\frac{U_\infty}{\nu x}}}$. Therefore;

$$\psi = U_\infty\sqrt{\frac{\nu x}{U_\infty}} f(\eta) = \sqrt{\nu x U_\infty} f(\eta), \tag{3.1.8}$$

where $\int g(\eta)d\eta = f(\eta)$ and it denotes the dimensionless stream function. Introducing the co-ordinate transformation, $(x, y) \mapsto (x, \eta)$, it follows from basic calculus that transition from the co-ordinates (x, y) to the coordinates (a, b), we have

$$\frac{\partial}{\partial x} = \frac{\partial a}{\partial x}\frac{\partial}{\partial a} + \frac{\partial b}{\partial x}\frac{\partial}{\partial b}$$

and

$$\frac{\partial}{\partial y} = \frac{\partial a}{\partial y}\frac{\partial}{\partial a} + \frac{\partial b}{\partial y}\frac{\partial}{\partial b}.$$

hence, for our case moving from (x, y) to (x, η), we then have with respect to x

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta}$$

and with respect to y, yields

$$\frac{\partial}{\partial y} = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}$$

Now, from the stream function (3.1.8), we can then find the velocity components in (x, y) co-ordinates as follows;

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ &= \frac{\partial \psi}{\partial \eta}\cdot\frac{\partial \eta}{\partial y} \\ &= \left[U_\infty\sqrt{\frac{\nu x}{U_\infty}}\frac{df}{d\eta}\cdot\sqrt{\frac{U_\infty}{\nu x}}\right] \end{aligned}$$

$$u = U_\infty f'(\eta) \tag{3.1.9}$$

Similarly, the transverse velocity component is given by

$$v = -\frac{\partial \psi}{\partial x} = -\left[\frac{\partial \psi}{\partial x} + \left(\frac{\partial \psi}{\partial \eta}\cdot\frac{\partial \eta}{\partial x}\right)\right] \tag{3.1.10}$$

From equation (3.1.7) and (3.1.8), $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ and $\psi = U_\infty\sqrt{\frac{\nu x}{U_\infty}}f(\eta)$, it then follows that the differential of ψ with respect to η becomes

$$\frac{\partial\psi}{\partial\eta} = \frac{\partial}{\partial\eta}\left[U_\infty\sqrt{\frac{\nu x}{U_\infty}}f(\eta)\right]$$

$$\frac{\partial\psi}{\partial\eta} = U_\infty\sqrt{\frac{\nu x}{U_\infty}}f'(\eta) \qquad (3.1.11)$$

And differentiating η with respect to x, yields

$$\begin{aligned}\frac{\partial\eta}{\partial x} &= \frac{\partial}{\partial x}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right] \\ &= y\sqrt{\frac{U_\infty}{\nu}}\frac{\partial}{\partial x}\left(x^{-\frac{1}{2}}\right)\end{aligned}$$

$$\frac{\partial\eta}{\partial x} = -\frac{\eta}{2x} = \eta_x \qquad (3.1.12)$$

Finally, differentiating ψ with respect to x, we obtain

$$\begin{aligned}\frac{\partial\psi}{\partial x} &= \frac{\partial}{\partial x}\left[U_\infty\sqrt{\frac{\nu x}{U_\infty}}f(\eta)\right] \\ &= U_\infty\sqrt{\frac{\nu}{U_\infty}}f(\eta)\frac{\partial}{\partial x}\left(x^{\frac{1}{2}}\right) \\ &= \frac{U_\infty}{2}\sqrt{\frac{\nu}{U_\infty x}}f(\eta)\end{aligned}$$

$$\frac{\partial\psi}{\partial x} = \frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}f(\eta) \qquad (3.1.13)$$

Thus substituting equation (3.1.11,3.1.12) and (3.1.13) in transverse velocity equation (3.1.10), yields

$$\begin{aligned} v &= -\left[\left(\frac{U_\infty}{2}\sqrt{\frac{\nu}{U_\infty x}}f(\eta)\right)+\left(U_\infty\sqrt{\frac{\nu x}{U_\infty}}\frac{df}{d\eta}\cdot\left(\frac{-\eta}{2x}\right)\right)\right] \\ &= \left(\frac{\eta f'}{2x}\sqrt{\nu U_\infty x}\right)-\left(\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}f(\eta)\right) \\ &= \frac{\eta f'(\eta)}{2}\sqrt{\frac{\nu U_\infty}{x}}-\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}f(\eta) \\ &= \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\left(\eta\frac{df}{d\eta}-f\right) \end{aligned}$$

Thus,

$$v=\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\left(\eta f'-f\right). \tag{3.1.14}$$

Also, differentiating the equations, (3.1.9) and (3.1.14) with respect to x and y respectively we get,

$$\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left[U_\infty\frac{df}{d\eta}\right],$$

where $f(\eta)=\frac{\psi}{U_\infty\sqrt{\frac{\nu x}{U_\infty}}}$. Now

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial \eta}\cdot\frac{\partial \eta}{\partial x}=\frac{\partial}{\partial \eta}\left[U_\infty\frac{df}{d\eta}\right]\cdot\frac{\partial}{\partial x}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right],$$

and therefore,

$$\left(U_\infty\frac{d^2 f}{d\eta^2}\right)\cdot\left(-\frac{y}{2x}\sqrt{\frac{U_\infty}{\nu x}}\right)=\left(U_\infty\frac{d^2 f}{d\eta^2}\right)\cdot\left(-\frac{\eta}{2x}\right).$$

Hence,

$$\frac{\partial u}{\partial x}=\frac{-\eta U_\infty}{2x}\frac{d^2 f}{d\eta^2}. \tag{3.1.15}$$

Similarly,

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}\left[U_\infty\frac{df}{d\eta}\right] \\ &= U_\infty f''(\eta)\frac{\partial \eta}{\partial y} \\ &= U_\infty f''(\eta)\sqrt{\frac{U_\infty}{\nu x}} \end{aligned}$$

Hence

$$\frac{\partial u}{\partial y} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} \frac{d^2 f}{d\eta^2} \tag{3.1.16}$$

Finally, the second derivative of u with respect to y becomes

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) \\ &= \frac{\partial}{\partial y}\left[U_\infty \sqrt{\frac{U_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}\right] \end{aligned}$$

Since $\frac{\partial}{\partial y} = \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$, the second differential becomes

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial \eta}\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}\right) \cdot \frac{\partial \eta}{\partial y} \\ &= \left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} f'''\right) \cdot \frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right) \\ &= \left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} f'''\right) \cdot \left(\sqrt{\frac{U_\infty}{\nu x}}\right) \end{aligned}$$

Thus

$$\frac{\partial^2 u}{\partial y^2} = \frac{U_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3}. \tag{3.1.17}$$

Substituting equations (3.1.9, 3.1.14, 3.1.15, 3.1.16) and (3.1.17) into the simple momentum equation (3.1.2) given by

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2},$$

we have

$$U_\infty f'\left(\frac{-U_\infty}{2x}\eta f''\right) + \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\,(\eta f' - f)\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} f''\right) = \nu \frac{U_\infty^2}{\nu x}\frac{d^3 f}{d\eta^3},$$

which implies that

$$\frac{-U_\infty^2}{2x}\eta f' f'' + \frac{1}{2}\eta f' f''\left(\sqrt{\frac{U_\infty \nu}{x}}\right)\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}}\right) - \frac{1}{2} f f''\left(\sqrt{\frac{U_\infty \nu}{x}}\right)\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}}\right) = \frac{U_\infty^2}{x} f''',$$

and thus

$$\frac{-U_\infty^2}{2x}\eta f'f'' + \frac{U_\infty^2}{2x}\eta f'f'' - \frac{1}{2}ff''\frac{U_\infty^2}{x} = \frac{U_\infty^2}{x}f'''.$$

Simplifying, we get

$$2f''' + ff'' = 0$$

$$f''' + \frac{1}{2}ff'' = 0 \tag{3.1.18}$$

with boundary conditions;

$$f(0) = 0, f'(0) = 0.$$

Furthermore, because $u \to U_\infty \quad as \quad y \to \infty$, $f'(\infty) = 1$. Equation (3.1.18) is a third order non-linear ordinary differential equation. Hence by similarity technique, the system of two Partial differential equations (PDEs) is converted to one Ordinary differential equation (ODE).

3.2 Problem Development

Consider the heat and mass transfer of a steady two-dimensional laminar flow of a viscous, incompressible, electrically conducting and dissipating fluid moving past an inclined infinite plate. The motion is in the presence of a uniform magnetic field of intensity B_0 applied normal to the plate surface. Assume the x axis of a cartesian coordinate system (x, y) is directed along the plate and the y axis is perpendicular to the plate surface. Then the origin of the coordinate system is taken to be the leading edge of the plate. The acceleration due to gravity g is taken to be acting vertically downwards. The plate surface is inclined to the vertical direction by an angle γ.

We assume that the fluid property variations due to temperature and chemical species concentration are limited to fluid density. Initially, the plate and the fluid are at same temperature T_∞ with concentration level C_∞ at all points. At time $t > 0$, the plate temperature is raised to T_w and a periodic temperature variation is assumed to be superimposed on this mean constant temperature of the plate and the concentration level at the plate is raised to C_w . In the analysis, we assume that the magnetic Reynolds number is much less than unity (it is very small for most fluids used in industrial applications) so that the induced magnetic field is neglected in comparison to the applied magnetic field. We shall neglect the Soret and Dufour effects as in [1] since we assume that the fluid under consideration has very small concentration of diffusing species in comparison to other chemical species. Thus, the concentration of species far from the plate wall, i.e. C_∞ is infinitesimally small.

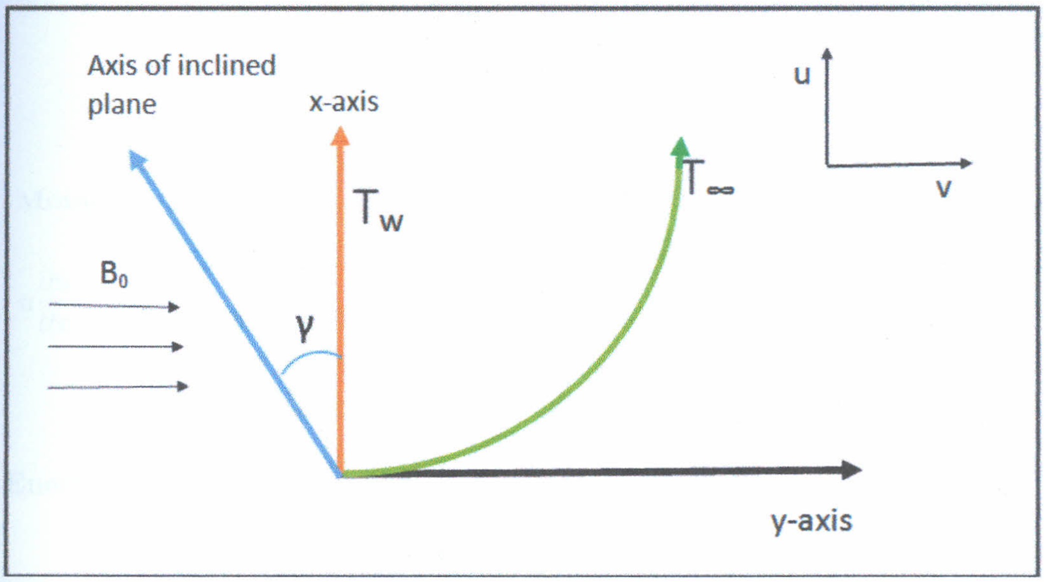

Figure 3.2.1: Physical configuration and coordinate system

Let u and v be the velocity components in the x and y axes directions respectively. Then, under the usual Boussinesqs and boundary layer approximations, the steady, laminar, two dimensional boundary layer flow under consideration can be governed by the following set of equations of continuity, momentum, energy and species concentration respectively as follows:

3.3 Governing Equations of the study

1. **Continuity Equation**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.3.1}$$

2. **Momentum Equation**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+g\beta(T-T_{\infty})\cos\gamma+g\beta^{*}(C-C_{\infty})\cos\gamma-\frac{\sigma_c B_0^2}{\rho}u \tag{3.3.2}$$

3. **Energy Equation**

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.3.3}$$

4. **Species Concentration Equation**

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{3.3.4}$$

where T is the temperature of the fluid in the boundary layer, T_{∞} is the temperature of the uniform flow far away from the plate, C_{∞} is species concentration in the fluid far away from the plate, β^{*} is the volumetric coefficient of expansion due to concentration, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, g is the gravitational acceleration, $\frac{k}{\rho C_p}$ is thermal diffusivity, D_m is the chemical species diffusivity coefficient, σ_c is the electrical conductivity, ρ is the density, $\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$ is viscous dissipation term, and B_0 is magnetic field intensity, and other variables and related quantities are defined in the index of notation with the following initial and boundary conditions. On the body surface, there is no slip condition and no permeation; hence,

$$u(x,0)=v(x,0)=0,\quad T=T_w+\in(T_w-T_{\infty})\cos\omega t,\quad C=C_w\quad \text{at}\quad y=0,t>0$$

At the exterior edge of the boundary $y\rightarrow\infty$, velocity must match the surface slipping velocity (free stream condition at infinity), that is,

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty, t > 0,$$

where T_w is the wall plate temperature and C_w is the chemical species concentration at the plate surface and $(T_w - T_\infty)\cos \omega t$ being the periodic temperature term.

3.4 Similarity Transformation Technique

We now introduce a two dimensional stream function ψ related to the velocities u and v according to the equations $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ so that continuity equation (3.3.1) is automatically satisfied as shown below. Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, then substituting in continuity equation (3.3.1), we have

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0.$$

Let η be the independent variable, expressed as $(\frac{y}{\delta})$, we then assume that the velocity may be expressed as a function of this variable to yield

$$\frac{u}{U_\infty} = g\left(\frac{y}{\delta}\right) = g(\eta).$$

It follows that

$$\eta \sim \frac{y}{\delta} = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} = y\sqrt{\frac{U_\infty}{\nu x}},$$

where δ is given as in (3.1.6). We call η the similarity variable and $g(\eta)$ is the function we seek as a solution.

We therefore introduce, the following local similarity variables

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}, \quad \psi(x,y) = \sqrt{\nu U_\infty x} f(\eta), \quad u = U_\infty f'(\eta), v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f'(\eta) - f(\eta)),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{and} \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{3.4.1}$$

where η is a similarity variable, $\theta(\eta)$ and $\phi(\eta)$ are the dimensionless temperature and concentration respectively, U_∞ is the velocity of the fluid far away from the plate. We then compute the partial derivatives in the governing equations. Now from the momentum equation (3.3.2), we compute the partial derivatives as follows

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial x \partial y} = \left[\eta_x \eta_y \frac{\partial^2 \psi}{\partial \eta^2} + \eta_{xy}\frac{\partial \psi}{\partial \eta}\right]$$

where $u = \frac{\partial \psi}{\partial y}$, $\psi = U_\infty \sqrt{\frac{\nu x}{U_\infty}} f(\eta) = \sqrt{\nu x U_\infty} f(\eta)$ and $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$. Therefore

$$\begin{aligned}\eta_x &= \frac{\partial \eta}{\partial x} = \frac{\partial}{\partial x}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right), \\ &= y\sqrt{\frac{U_\infty}{\nu}}\frac{\partial}{\partial x}\left(x^{-\frac{1}{2}}\right),\end{aligned}$$

hence

$$\eta_x = -\frac{y}{2x}\sqrt{\frac{U_\infty}{\nu x}} = -\frac{\eta}{2x}.$$

And

$$\eta_y = \frac{\partial \eta}{\partial y} = \frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right) = \sqrt{\frac{U_\infty}{\nu x}}.$$

Then $\frac{\partial^2 \psi}{\partial \eta^2} = \frac{\partial}{\partial \eta}\left(\frac{\partial \psi}{\partial \eta}\right)$, but

$$\frac{\partial \psi}{\partial \eta} = \frac{\partial}{\partial \eta}\left(U_\infty \sqrt{\frac{\nu x}{U_\infty}} f(\eta)\right) = U_\infty \sqrt{\frac{\nu x}{U_\infty}} f'(\eta).$$

Therefore the second differential of ψ with respect to η becomes,

$$\begin{aligned}\frac{\partial^2 \psi}{\partial \eta^2} &= \frac{\partial}{\partial \eta}\left(U_\infty \sqrt{\frac{\nu x}{U_\infty}} f'(\eta)\right), \\ &= U_\infty \sqrt{\frac{\nu x}{U_\infty}} f''(\eta).\end{aligned}$$

It follows that

$$\begin{aligned}\eta_x \eta_y \frac{\partial^2 \psi}{\partial \eta^2} &= \left(-\frac{\eta}{2x}\right)\left(\sqrt{\frac{U_\infty}{\nu x}}\right)\left(U_\infty \sqrt{\frac{\nu x}{U_\infty}} f''(\eta)\right), \\ &= -\frac{\eta U_\infty}{2x} f''(\eta).\end{aligned}$$

Again

$$\eta_{xy} = \frac{\partial^2 \eta}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial \eta}{\partial x}\right) = \frac{\partial}{\partial y}\left(-\frac{\eta}{2x}\right)$$

But since $\frac{\partial}{\partial y} = \left(\frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)$, it implies that η_{xy} reduces to

$$\begin{aligned}\eta_{xy} &= \frac{\partial}{\partial \eta}\left(-\frac{\eta}{2x}\right)\frac{\partial \eta}{\partial y}, \\ &= -\frac{1}{2x}\left[\frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right)\right],\end{aligned}$$

hence

$$\eta_{xy} = -\frac{1}{2x}\sqrt{\frac{U_\infty}{\nu x}}.$$

So that

$$\begin{aligned}\eta_{xy}\frac{\partial \psi}{\partial \eta} &= \left[-\frac{1}{2x}\sqrt{\frac{U_\infty}{\nu x}}\right]\left(U_\infty\sqrt{\frac{\nu x}{U_\infty}}f'(\eta)\right), \\ &= -\frac{U_\infty}{2x}f'(\eta).\end{aligned}$$

Therefore, the partial differential of u with respect to x now becomes

$$\begin{aligned}\frac{\partial u}{\partial x} &= -\frac{\eta U_\infty}{2x}f''(\eta) - \frac{U_\infty}{2x}f'(\eta), \\ &= -\frac{U_\infty}{2x}(f' + \eta f''),\end{aligned}$$

It follows that

$$u\frac{\partial u}{\partial x} = U_\infty f'(\eta)\left(-\frac{U_\infty}{2x}(f' + \eta f'')\right).$$

And therefore

$$u\frac{\partial u}{\partial x} = -\frac{U_\infty^2}{2x}(f'f' + \eta f'f'') \tag{3.4.2}$$

Similarly the partial differential of u with respect to y is given by

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial y^2}$$

But since $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$, it follows that

$$\begin{aligned}
\frac{\partial u}{\partial y} &= \frac{\partial}{\partial \eta}\left[U_\infty \frac{df}{d\eta}\right] \cdot \frac{\partial}{\partial y}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right], \\
&= \left(U_\infty \frac{d^2 f}{d\eta^2}\right) \cdot \left(\sqrt{\frac{U_\infty}{\nu x}}\right),
\end{aligned}$$

thus,

$$\frac{\partial u}{\partial y} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} f''(\eta).$$

Then,

$$\begin{aligned}
v\frac{\partial u}{\partial y} &= \left[\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f'(\eta) - f)\right]\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} f''\right), \\
&= \frac{U_\infty^2}{2x}(\eta f' - f) f''(\eta),
\end{aligned}$$

and hence

$$v\frac{\partial u}{\partial y} = \frac{U_\infty^2}{2x}(\eta f' f'' - f f''). \tag{3.4.3}$$

Again the second differential of u with respect to y yields

$$\begin{aligned}
\frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right], \\
&= \frac{\partial}{\partial y}\left(\frac{\partial^2 \psi}{\partial y^2}\right) = \frac{\partial^3 \psi}{\partial y^3}, \\
&= \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(U_\infty \sqrt{\frac{U_\infty}{\nu x}} f''(\eta)\right),
\end{aligned}$$

but since $\frac{\partial}{\partial y} = \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$, it follows that

$$\begin{aligned}
\frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial \eta}\left[U_\infty \sqrt{\frac{U_\infty}{\nu x}} f''(\eta)\right] \cdot \frac{\partial \eta}{\partial y}, \\
&= U_\infty \sqrt{\frac{U_\infty}{\nu x}} f'''(\eta) \cdot \frac{\partial}{\partial y}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right], \\
&= U_\infty \sqrt{\frac{U_\infty}{\nu x}} f'''(\eta) \cdot \sqrt{\frac{U_\infty}{\nu x}}.
\end{aligned}$$

Therefore

$$\frac{\partial^2 u}{\partial y^2} = \frac{U^2\infty}{\nu x} f'''(\eta),$$

and thus

$$\nu\frac{\partial^2 u}{\partial y^2} = \frac{U^2\infty}{x} f'''(\eta). \tag{3.4.4}$$

The next set of transformation is the energy equation. The attempt is once again made to see if the energy equation can be transformed into one where the governing equation becomes an ordinary differential equation. Therefore, we introduce a non-dimensional temperature of the form

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}.$$

And since T_∞ is constant, we have $\theta(T_w - T_\infty) = T - T_\infty$, and therefore

$$\begin{aligned}\frac{\partial T}{\partial x} &= (T_w - T_\infty)\frac{\partial \theta}{\partial x} + \theta\frac{\partial}{\partial x}(T_w - T_\infty) + \frac{\partial T_\infty}{\partial x},\\ &= (T_w - T_\infty)\frac{\partial \theta}{\partial x}.\end{aligned}$$

But $\frac{\partial\theta}{\partial x} = \frac{d\theta}{d\eta}\cdot\frac{\partial\eta}{\partial x}$, let $\frac{d\theta}{d\eta} = \theta'$, then substituting in $\frac{\partial T}{\partial x}$, we obtain

$$\begin{aligned}\frac{\partial T}{\partial x} &= \theta'(T_w - T_\infty)\frac{\partial}{\partial x}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right],\\ &= \theta'(T_w - T_\infty)y\sqrt{\frac{U_\infty}{\nu}}\left[\frac{\partial}{\partial x}\left(x^{-\frac{1}{2}}\right)\right],\\ &= -\frac{y}{2x}\sqrt{\frac{U_\infty}{\nu x}}\theta'(T_w - T_\infty) = -\frac{\eta}{2x}(T_w - T_\infty)\theta'.\end{aligned}$$

Then since $u = U_\infty f'(\eta)$, we have

$$\begin{aligned}u\frac{\partial T}{\partial x} &= U_\infty f'(\eta)\left(\frac{\partial T}{\partial x}\right),\\ &= U_\infty f'(\eta)\left[-\frac{\eta}{2x}(T_w - T_\infty)\theta'\right],\end{aligned}$$

Hence

$$u\frac{\partial T}{\partial x} = -\frac{\eta U_\infty}{2x}(T_w - T_\infty)\theta' f'(\eta). \tag{3.4.5}$$

Similarly,

$$\begin{aligned}\frac{\partial T}{\partial y} &= (T_w - T_\infty)\frac{\partial \theta}{\partial y} + \theta\frac{\partial}{\partial y}(T_w - T_\infty) + \frac{\partial}{\partial y}(T_\infty),\\ &= (T_w - T_\infty)\frac{\partial \theta}{\partial y}.\end{aligned}$$

And since $\frac{\partial \theta}{\partial y} = \frac{d\theta}{d\eta} \cdot \frac{\partial \eta}{\partial y}$, it follows that

$$\begin{aligned}\frac{\partial T}{\partial y} &= (T_w - T_\infty)\theta'\frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right),\\ &= \sqrt{\frac{U_\infty}{\nu x}}(T_w - T_\infty)\theta'.\end{aligned}$$

Since $v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f'(\eta) - f(\eta))$, we have

$$v\frac{\partial T}{\partial y} = \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(\eta f' - f)\left[\sqrt{\frac{U_\infty}{\nu x}}(T_w - T_\infty)\theta'\right],$$

therefore,

$$v\frac{\partial T}{\partial y} = \frac{U_\infty}{2x}(T_w - T_\infty)\theta'\,(\eta f'(\eta) - f(\eta)). \tag{3.4.6}$$

The first term on the right hand side of energy equation (3.3.3), that is $\frac{\partial^2 T}{\partial y^2}$, then is transformed as follows

$$\begin{aligned}\frac{\partial^2 T}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right),\\ &= \frac{\partial}{\partial y}\left[(T_w - T_\infty)\frac{\partial \theta}{\partial y}\right],\\ &= (T_w - T_\infty)\frac{\partial}{\partial y}\left(\frac{d\theta}{d\eta}\cdot\frac{\partial \eta}{\partial y}\right),\\ &= (T_w - T_\infty)\frac{\partial}{\partial y}\left(\frac{d\theta}{d\eta}\sqrt{\frac{U_\infty}{\nu x}}\right).\end{aligned}$$

On substituting $\frac{\partial}{\partial y} = \left(\frac{d}{d\eta} \cdot \frac{\partial \eta}{\partial y}\right)$, we obtain

$$\begin{aligned}
\frac{\partial^2 T}{\partial y^2} &= (T_w - T_\infty)\sqrt{\frac{U_\infty}{\nu x}} \frac{d}{d\eta}\left(\frac{d\theta}{d\eta}\right)\frac{\partial \eta}{\partial y}, \\
&= (T_w - T_\infty)\left[\sqrt{\frac{U_\infty}{\nu x}}\right]^2 \frac{d}{d\eta}\left(\frac{d\theta}{d\eta}\right), \\
&= \frac{U_\infty}{\nu x}(T_w - T_\infty)\theta''.
\end{aligned}$$

Hence

$$\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} = \frac{k}{\rho C_p}\frac{U_\infty}{\nu x}(T_w - T_\infty)\theta''. \tag{3.4.7}$$

The last term on the right hand side of equation (3.3.3), represents the viscous dissipation term, and is transformed as below

$$\begin{aligned}
\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 &= \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)^2, \\
&= \frac{\mu}{\rho C_p}\left[\frac{\partial}{\partial \eta}(U_\infty f'(\eta)) \cdot \frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right)\right]^2, \\
&= \frac{\mu}{\rho C_p}\left[U_\infty f''(\eta) \cdot \sqrt{\frac{U_\infty}{\nu x}}\right]^2.
\end{aligned}$$

Thus

$$\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 = \frac{\mu}{\rho C_p}\frac{U_\infty}{\nu x}\left[U_\infty f''(\eta)\right]^2. \tag{3.4.8}$$

Finally, for concentration equation (3.3.4), we similarly introduce the non-dimensional concentration

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}.$$

Then $\phi(C_w - C_\infty) = C - C_\infty$, and so

$$\begin{aligned}
\frac{\partial C}{\partial x} &= (C_w - C_\infty)\frac{\partial \phi}{\partial x} + \phi\frac{\partial}{\partial x}(C_w - C_\infty) + \frac{\partial}{\partial x}(C_\infty), \\
&= (C_w - C_\infty)\frac{\partial \phi}{\partial x}.
\end{aligned}$$

But $\frac{\partial \phi}{\partial x} = \left(\frac{d\phi}{d\eta} \cdot \frac{\partial \eta}{\partial x}\right)$ and let $\frac{d\phi}{d\eta} = \phi'$, to get

$$\begin{aligned}
\frac{\partial C}{\partial x} &= (C_w - C_\infty)\left(\frac{d\phi}{d\eta} \cdot \frac{\partial \eta}{\partial x}\right), \\
&= (C_w - C_\infty)\phi' \frac{\partial}{\partial x}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right], \\
&= (C_w - C_\infty)\phi' y\sqrt{\frac{U_\infty}{\nu}} \frac{\partial}{\partial x}(x^{-\frac{1}{2}}), \\
&= -(C_w - C_\infty)\phi' \frac{y}{2x}\sqrt{\frac{U_\infty}{\nu x}}, \\
&= -\frac{\eta}{2x}(C_w - C_\infty)\phi'.
\end{aligned}$$

Since $u = U_\infty f'(\eta)$, we have

$$u\frac{\partial C}{\partial x} = -\frac{\eta}{2x}(C_w - C_\infty)\phi' \cdot (U_\infty f'(\eta)).$$

It follows that,

$$u\frac{\partial C}{\partial x} = -\frac{\eta U_\infty}{2x}(C_w - C_\infty)\phi' f'(\eta). \tag{3.4.9}$$

Similarly

$$\frac{\partial C}{\partial y} = (C_w - C_\infty)\frac{\partial \phi}{\partial y}.$$

And by chain rule $\frac{\partial \phi}{\partial y} = \left(\frac{d\phi}{d\eta} \cdot \frac{\partial \eta}{\partial y}\right)$, now let $\frac{d\phi}{d\eta} = \phi'$, substituting in $\frac{\partial C}{\partial y}$, we get

$$\begin{aligned}
\frac{\partial C}{\partial y} &= (C_w - C_\infty)\left(\frac{d\phi}{d\eta} \cdot \frac{\partial \eta}{\partial y}\right), \\
&= (C_w - C_\infty)\phi' \frac{\partial}{\partial y}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right), \\
&= (C_w - C_\infty)\phi'\sqrt{\frac{U_\infty}{\nu x}}, \\
&= \sqrt{\frac{U_\infty}{\nu x}}(C_w - C_\infty)\phi'.
\end{aligned}$$

Since $v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left(\eta f'(\eta) - f(\eta)\right)$, it follows that

$$
\begin{aligned}
v\frac{\partial C}{\partial y} &= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left(\eta f'(\eta) - f(\eta)\right)\left[\sqrt{\frac{U_\infty}{\nu x}}(C_w - C_\infty)\phi'\right], \\
&= \frac{1}{2}\frac{U_\infty}{x}(C_w - C_\infty)\left[\eta f'(\eta) - f(\eta)\right]\phi'.
\end{aligned}
$$

Thus

$$
v\frac{\partial C}{\partial y} = \frac{U_\infty}{2x}(C_w - C_\infty)\left[\eta f'(\eta) - f(\eta)\right]\phi'. \tag{3.4.10}
$$

The last term in the species equation (3.3.4) becomes

$$
\begin{aligned}
\frac{\partial^2 C}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial C}{\partial y}\right) = \left((C_w - C_\infty)\frac{\partial \phi}{\partial y}\right)\frac{\partial}{\partial y}, \\
&= (C_w - C_\infty)\left(\frac{d\phi}{d\eta}\cdot\frac{\partial \eta}{\partial y}\right)\frac{\partial}{\partial y}, \\
&= (C_w - C_\infty)\frac{\partial}{\partial y}\left(\frac{d\phi}{\partial \eta}\cdot\frac{\partial}{\partial y}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right]\right), \\
&= (C_w - C_\infty)\frac{\partial}{\partial y}\left(\frac{d\phi}{d\eta}\cdot\sqrt{\frac{U_\infty}{\nu x}}\right),
\end{aligned}
$$

On substituting $\frac{\partial}{\partial y} = \frac{d}{d\eta}\cdot\frac{\partial \eta}{\partial y}$, we obtain

$$
\begin{aligned}
\frac{\partial^2 C}{\partial y^2} &= (C_w - C_\infty)\sqrt{\frac{U_\infty}{\nu x}}\frac{d}{d\eta}\left(\frac{d\phi}{d\eta}\right)\frac{\partial \eta}{\partial y}, \\
&= (C_w - C_\infty)\sqrt{\frac{U_\infty}{\nu x}}\phi''\frac{\partial}{\partial y}\left[y\sqrt{\frac{U_\infty}{\nu x}}\right], \\
&= \frac{U_\infty}{\nu x}(C_w - C_\infty)\phi''.
\end{aligned}
$$

Therefore

$$
D_m\frac{\partial^2 C}{\partial y^2} = D_m\frac{U_\infty}{\nu x}(C_w - C_\infty)\phi''. \tag{3.4.11}
$$

Substituting the equations numbered (3.4.1 - 3.4.4) into momentum equation (3.3.2), given as

$$
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty)\cos\gamma + g\beta^*(C - C_\infty)\cos\gamma - \frac{\sigma_c B_0^2}{\rho}u,
$$

we have

$$(U_\infty f'(\eta))\left[-\frac{U_\infty}{2x}(\eta f''+f')\right]+\left(\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(\eta f'-f)\right)U_\infty\sqrt{\frac{U_\infty}{\nu x}}f''(\eta)=\frac{\mu}{\rho}\left(\frac{U_\infty^2}{\nu x}f'''(\eta)\right)$$

$$+g\beta(T-T_\infty)\cos\gamma+g\beta^*(C-C_\infty)\cos\gamma-\frac{\sigma_c B_o^2}{\rho}u,$$

$$\Rightarrow -\frac{U_\infty^2}{2x}(\eta f'f''+f'f')+\frac{1}{2}\eta f'f''\left(\sqrt{\frac{U_\infty \nu}{x}}\right)\left(U_\infty\sqrt{\frac{U_\infty}{\nu x}}\right)-\frac{1}{2}ff''\left(\sqrt{\frac{U_\infty \nu}{x}}\right)\left(U_\infty\sqrt{\frac{U_\infty}{\nu x}}\right)$$

$$=\frac{\mu}{\rho}\frac{U_\infty^2}{\nu x}f'''(\eta)+g\beta(T-T_\infty)\cos\gamma+g\beta^*(C-C_\infty)\cos\gamma-\frac{\sigma_c B_o^2}{\rho}(U_\infty f'(\eta)),$$

$$\Rightarrow -\frac{U_\infty^2}{2x}(\eta f'f''+f'f')+\frac{1}{2}\eta f'f''\left(\frac{U_\infty^2}{x}\right)-\frac{1}{2}ff''\left(\frac{U_\infty^2}{x}\right)=\frac{\mu}{\rho}\frac{U_\infty^2}{\nu x}f'''(\eta)+g\beta(T-T_\infty)\cos\gamma+$$

$$g\beta^*(C-C_\infty)\cos\gamma-\frac{\sigma_c B_o^2}{\rho}(U_\infty f'(\eta)),$$

Multiplying through by $\frac{x}{U_\infty^2}$ and let $\frac{\mu}{\rho}=\nu$, we have

$$-\frac{1}{2}\eta f'f''-\frac{1}{2}f'f'+\frac{1}{2}\eta f'f''-\frac{1}{2}ff''=f'''(\eta)+g\beta(T-T_\infty)\cos\gamma\left(\frac{x}{U_\infty^2}\right)+$$

$$g\beta^*(C-C_\infty)\cos\gamma\left(\frac{x}{U_\infty^2}\right)-\frac{\sigma_c B_o^2}{\rho}(U_\infty f'(\eta))\left(\frac{x}{U_\infty^2}\right),$$

$$\Rightarrow -\frac{1}{2}\eta f'f''-\frac{1}{2}f'f'+\frac{1}{2}\eta f'f''-\frac{1}{2}ff''=f'''(\eta)+\frac{g\beta x(T-T_\infty)\cos\gamma}{U_\infty^2}+\frac{g\beta^* x(C-C_\infty)\cos\gamma}{U_\infty^2}$$

$$-\frac{\sigma_c B_o^2 x}{\rho U_\infty}f'(\eta), \tag{3.4.12}$$

From the dimensionless temperature

$$\theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},$$

we have that

$$T-T_\infty=\theta(T_w-T_\infty). \tag{3.4.13}$$

And also from the dimensionless concentration

$$\phi(\eta)=\frac{(C-C_\infty)}{(C_w-C_\infty)},$$

we have

$$C - C_\infty = \phi(C_w - C_\infty), \tag{3.4.14}$$

Substituting dimensionless temperature equation (3.4.13) and dimensionless concentration equation (3.4.14) in equation (3.4.12), we get

$$-\frac{1}{2}f'f' - \frac{1}{2}ff'' = f'''(\eta) + \frac{g\beta x\theta(T_w - T_\infty)\cos\gamma}{U_\infty^2} + \frac{g\beta^* x\phi(C_w - C_\infty)\cos\gamma}{U_\infty^2} - \frac{\sigma_c B_o^2 x f'(\eta)}{\rho U_\infty}, \tag{3.4.15}$$

Now let

$$Gr = \frac{g\beta x(T_w - T_\infty)}{U_\infty^2}, \quad Gc = \frac{g\beta^* x(C_w - C_\infty)}{U_\infty^2} \quad \text{and} \quad M = \frac{\sigma_c B_0^2 x}{\rho U_\infty}$$

Substituting these parameters in equation (3.4.15), we obtain

$$-\frac{1}{2}f'f' - \frac{1}{2}ff'' = f'''(\eta) + \theta Gr\cos\gamma + \phi Gc\cos\gamma - Mf'(\eta)$$

Rearranging we have

$$f''' + \frac{1}{2}ff'' + \frac{1}{2}(f')^2 + \theta Gr\cos\gamma + \phi Gc\cos\gamma - Mf' = 0 \tag{3.4.16}$$

Similarly, consider the energy equation (3.3.3), given as,

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$$

Substituting equations numbered (3.4.1) and (3.4.5),(3.4.6),(3.4.7) and (3.4.8) in the energy equation (3.3.3), we have

$$-\frac{\eta U_\infty}{2x}(T_w - T_\infty)\theta' f'(\eta) + \frac{U_\infty}{2x}(T_w - T_\infty)\theta'\left(\eta f'(\eta) - f(\eta)\right) = \frac{k}{\rho C_p}\frac{U_\infty}{\nu x}(T_w - T_\infty)\theta'' +$$

$$\frac{\mu}{\rho C_p}\frac{U_\infty}{\nu x}\left(U_\infty f''(\eta)\right)^2,$$

Dividing through by $(T_w - T_\infty)$, we get

$$-\frac{\eta U_\infty}{2x}\theta' f'(\eta) + \frac{U_\infty}{2x}\theta'\left(\eta f'(\eta) - f(\eta)\right) = \frac{k}{\rho C_p}\frac{U_\infty}{\nu x}\theta'' + \frac{\mu}{\rho C_p}\frac{U_\infty}{\nu x}\frac{\left(U_\infty f''(\eta)\right)^2}{(T_w - T_\infty)},$$

Now multiplying through by $\frac{x}{U_\infty}$, we arrive at

$$-\frac{\eta}{2}\theta' f'(\eta) + \frac{1}{2}\theta'\left(\eta f'(\eta) - f(\eta)\right) = \frac{k}{\rho C_p \nu}\theta'' + \frac{\mu}{\rho C_p \nu}\frac{U_\infty^2 (f'')^2}{(T_w - T_\infty)}.$$

Simplifying, we have

$$-\frac{1}{2}\theta' f = \frac{k}{\rho C_p \nu}\theta'' + \frac{\mu}{\rho \nu}\frac{U_\infty^2}{C_p(T_w - T_\infty)}(f'')^2,$$

Therefore, multiplying through by $\frac{\rho C_p \nu}{k}$, yields

$$\theta'' + \frac{1}{2}\theta' f\left(\frac{\rho C_p \nu}{k}\right) + \frac{\mu}{\rho}\frac{U_\infty^2}{C_p(T_w - T_\infty)}(f'')^2 \cdot \left(\frac{\rho C_p}{k}\right). \qquad (3.4.17)$$

Let $\alpha = \frac{k}{\rho C_p}$, $Pr = \frac{\nu}{\alpha}$ and $Ec = \frac{U_\infty^2}{C_p(T_w - T_\infty)}$, then substituting in equation (3.4.17), we obtain

$$\theta'' + \frac{1}{2}\theta' f\left(\frac{\nu}{\alpha}\right) + Ec(f'')^2\frac{\mu}{\rho}\frac{1}{\alpha} = 0. \qquad (3.4.18)$$

Again let $\frac{\mu}{\rho} = \nu$, replacing in equation (3.4.18), it reduces to

$$\theta'' + \frac{1}{2}\theta' f Pr + Ec\left(\frac{\nu}{\alpha}\right)(f'')^2 = 0.$$

Hence, we get the transformed energy equation as

$$\theta'' + \frac{1}{2}Pr f\theta' + PrEc(f'')^2 = 0. \qquad (3.4.19)$$

Finally, consider the concentration equation (3.3.4) given as

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2}.$$

Now substituting the equations numbered (3.4.9),(3.4.10) and (3.4.11) into species equation (3.3.4), we obtain

$$-\frac{\eta U_\infty}{2x}(C_w - C_\infty)\phi' f'(\eta) + \frac{U_\infty}{2x}(C_w - C_\infty)\left[\eta f'(\eta) - f(\eta)\right]\phi' = D_m\frac{U_\infty}{\nu x}(C_w - C_\infty)\phi''.$$

Dividing through by $C_w - C_\infty$, we have

$$-\frac{\eta U_\infty}{2x}\phi' f'(\eta) + \frac{U_\infty}{2x}\left[\eta f'(\eta) - f(\eta)\right]\phi' = D_m \frac{U_\infty}{\nu x}\phi'',$$

simplifying, we get

$$-\frac{U_\infty}{2x} f\phi' = D_m \frac{U_\infty}{\nu x}\phi''$$

And multiplying through by $\frac{x}{U_\infty}$, we arrive at

$$-\frac{U_\infty}{2x}\left(\frac{x}{U_\infty}\right) f\phi' = D_m \frac{U_\infty}{\nu x}\left(\frac{x}{U_\infty}\right)\phi''$$

$$\Rightarrow -\frac{1}{2} f\phi' = \frac{D_m}{\nu}\phi''$$

Again multiplying through by $\frac{\nu}{D_m}$ gives

$$-\frac{1}{2} f\phi'\left(\frac{\nu}{D_m}\right) = \phi''.$$

Then let $\frac{\nu}{D_m} = Sc$, and rearranging, we get the transformed species concentration equation

$$\phi'' + \frac{1}{2} Scf\phi' = 0. \tag{3.4.20}$$

Thus, by similarity transformation, we find three non-linear ordinary differential equations:

$$f''' + \frac{1}{2} ff'' + \frac{1}{2}(f')^2 + \theta Gr\cos\gamma + \phi Gc\cos\gamma - Mf' = 0 \tag{3.4.21}$$

$$\theta'' + \frac{1}{2} Prf\theta' + PrEc(f'')^2 = 0 \tag{3.4.22}$$

$$\phi'' + \frac{1}{2} Scf\phi' = 0 \tag{3.4.23}$$

where the derivatives are taken with respect to η and

$$Gr = \frac{g\beta x(T_w - T_\infty)}{U_\infty^2}, \quad Gc = \frac{g\beta^* x(C_w - C_\infty)}{U_\infty^2}$$

$$Pr = \frac{\nu}{\alpha}, \quad E_c = \frac{U_\infty^2}{C_p(T_w - T_\infty)}, \quad M = \frac{\sigma_c B_0^2 x}{\rho U_\infty}, \quad Sc = \frac{\nu}{D_m}, \quad \alpha = \frac{k}{\rho C_p},$$

in which Gr is the local thermal Grashof number, Gc is the solutal or local concentration Grashof number, Sc is the Schimdt number and Pr is the Prandtl number, E_c is the Eckert number and α is the thermal diffusivity, M magnetic field parameter. The corresponding initial and boundary conditions are generated as follows, for the no slip condition at the wall, at $y = 0 \Rightarrow \eta = 0$, therefore, $f(\eta = 0) = 0$ and from $u = U_\infty f'(\eta) \Rightarrow f'(\eta) = \frac{u}{U_\infty}$, and at the wall $u = 0 \Rightarrow f' = 0$. Similarly, $T = T_w$ and $C = C_w$, hence, $\theta = \phi = 1$.
At the free stream condition, $y = \infty \Rightarrow \eta = \infty$, $T = T_\infty$ and $C = C_\infty$.
Thus, $f' \to 0, \theta \to 0, \phi \to 0$. Giving,

$$f = 0, \quad f' = 0, \quad \theta = 1, \quad \phi = 1 \quad at \ \eta = 0 \tag{3.4.24}$$

$$f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad as \ \eta \to \infty \tag{3.4.25}$$

CHAPTER FOUR

SIMULATION, RESULTS AND DISCUSSIONS

The similarity transformation converts the non-linear partial differential equations (3.3.1)-(3.3.4) into the set of nonlinear ordinary differential equations given by the set (3.4.21) - (3.4.23) with boundary conditions (3.4.24 - 3.4.25). This set of ODEs have been solved numerically using shooting method, a technique that converts the boundary value ordinary differential equations into a set of first order initial value ordinary differential equations. An over view to the numerical technique employed is that first, higher order nonlinear differential equations (3.4.21) - (3.4.23) are converted into simultaneous linear differential equations of first order and further transformed into initial value problem by applying the shooting technique. The resulting system is solved by the fourth-order Runge-Kutta method implemented in Mathematica to generate results.

In order to study the behaviour of velocity, temperature and concentration fields, a comprehensive numerical computation was carried out for various values of the parameters describing the flow characteristics. We assigned physically realistic numerical values to the embedded parameters in the system, to gain an insight into the flow structure with respect to velocity, temperature and species concentration profiles and the results reported in terms of graphs as presented below.

The Prandtl number was taken to be $Pr = 0.71$, which corresponds to air at 20^0c and 1 atmospheric pressure, electrolyte solution such as salt $Pr = 1.0$ and water $Pr = 7.0$. The values of Schmidt number (Sc) were chosen to be $Sc = 0.22, 0.60, 0.78, 0.96$, representing diffusing chemical species of most common interest in air like Hydrogen at 25^0c and 1 atmospheric pressure, Water vapour, Ammonia and Carbon dioxide respectively.

4.1 Effects of parameter variation on Velocity Profiles

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters like the magnetic field parameter M, Prandtl number Pr, Grashof number Gr, Solutal Grashof number Gc, Schmidt number Sc, Eckert number Ec, Angle of inclination γ on velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$. In the present study, the numerical values of different thermo physical parameters were specified as follows: $Gr = Gm = 0.05, \gamma = 30^0, M = 0.75, Pr = 0.71, Ec = 0.01, Sc = 0.6, \omega = t = 1.0, \epsilon = 1.0$ and adopted for computations. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

4.1.1 Effect of variation of Prandtl number on velocity

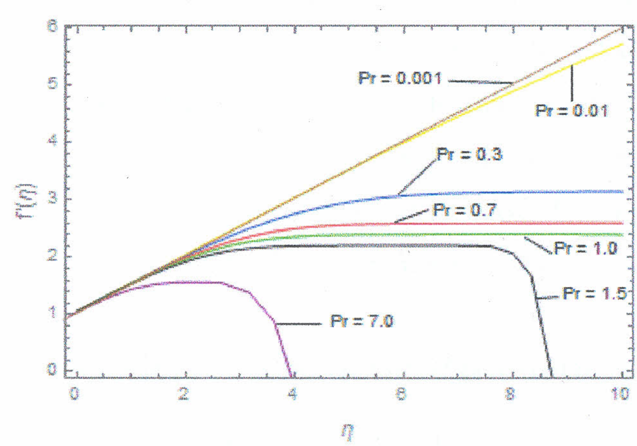

Figure 4.1.1: Velocity profiles for different values of Pr

Figure 4.1.1 shows the effect of variation of Prandtl number (Pr). The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in decreasing velocity. The velocity for $Pr = 0.71$ is higher than that of $Pr = 7.0$. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Also, for such low Prandtl number, the velocity boundary layer is inside the thermal boundary layer and its thickness (δ) reduces as Prandtl number decreases so the fluid motion is confined in more and more thinner layer near the surface.

4.1.2 Effect of variation of angle of Inclination on velocity

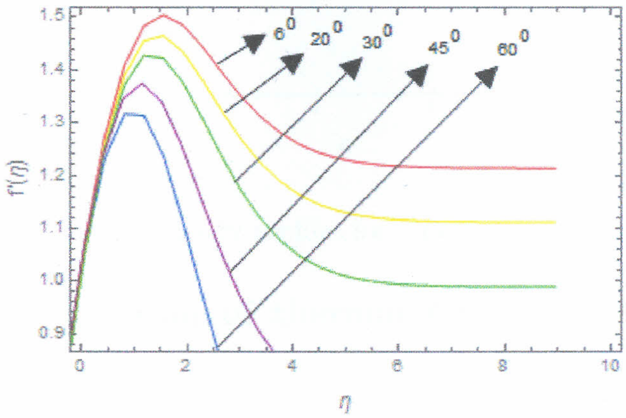

Figure 4.1.2: Velocity profiles for different values of γ

Figure 4.1.2 shows the effect of varying the inclination angle to the vertical direction on the velocity profiles. From this figure we observe that the velocity is decreased by increasing the angle of inclination γ. The fluid has higher velocity when the surface is vertical ($\gamma = 0$) than when inclined because of the fact that the buoyancy effect decreases due to gravity components ($g \cos \gamma$), as the plate is inclined. Consequently the driving force to the fluid decreases as a result velocity profiles decrease.

4.1.3 Effect of variation of Magnetic Parameter on velocity

For various values of the magnetic parameter M, the velocity profiles are plotted in Figure 4.1.3. An increase in magnetic field parameter, M, is observed to strongly reduce the velocity in the regime. Maximum velocity corresponds to $M = 0$ i.e. electrically non conducting heat and mass transfer. Physically, it is true due to the fact that the application of a transverse magnetic field to an electrically conducting fluid gives rise to a body force known as a Lorentz hydromagnetic drag which acts in the tangential direction. This force, $-(M)f'$, impedes the flow and reduces velocities i.e. decreases the hydrodynamic boundary layer thickness.

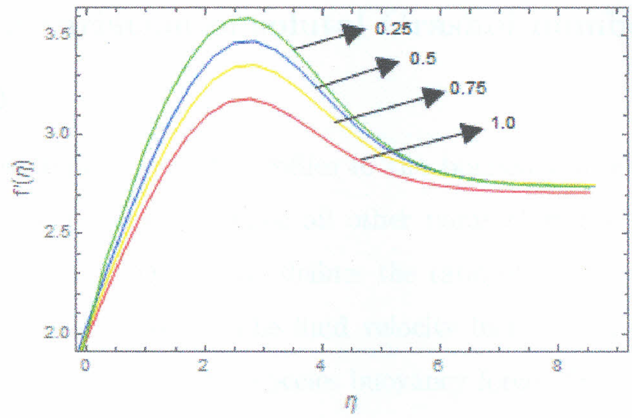

Figure 4.1.3: Velocity profiles for different values of M

4.1.4 Effect of variation of thermal Grashof (Gr) number on velocity

The influence of the thermal Grashof number on the velocity is presented in Fig. 4.1.4. The thermal Grashof number Gr is a measure of the relative magnitudes of the buoyancy force and the opposing viscous force acting on the fluid. From this graphical analysis, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the fluid velocity increases, reaching its peak value within the boundary layer and then decreases monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition.

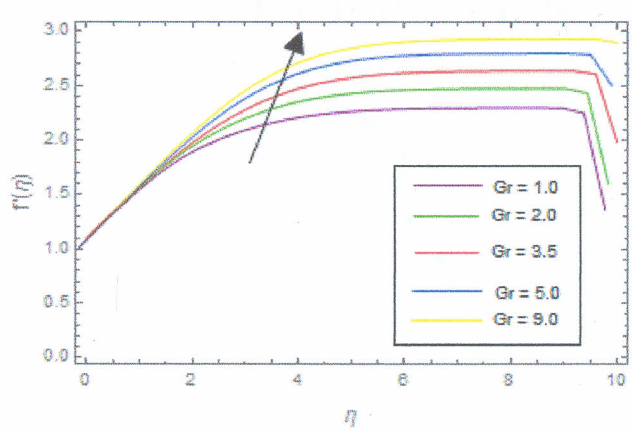

Figure 4.1.4: Velocity profiles for different values of Grashof (Gr)

4.1.5 Effect of variation of solutal Grashof number(Gc) on velocity

Figure 4.1.5 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gc, while all other parameters are kept at some fixed values. The solutal Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. The fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases to approach the free stream value.

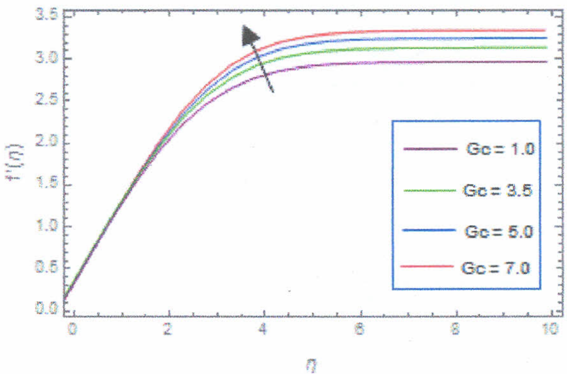

Figure 4.1.5: Velocity profiles for different values of solutal Grashof (Gc)

4.1.6 Effect of variation of Eckert (Ec) number on velocity

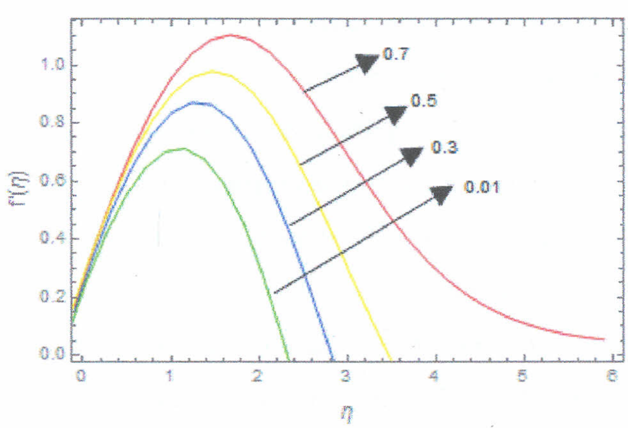

Figure 4.1.6: Velocity profiles for different values of Eckert (Ec)

The effect of the viscous dissipation parameter (Eckert number), is shown in figure 4.1.6. The Eckert number expresses the relationship between the kinetic energy of the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the velocity, which is evidenced in the above figure.

4.1.7 Effect of variation of Schmidt (Sc) number on velocity

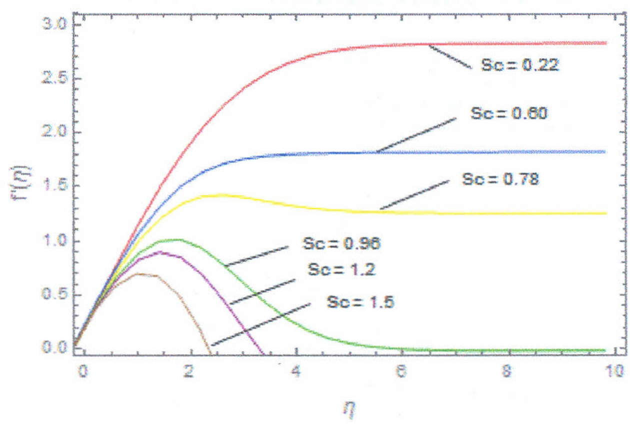

Figure 4.1.7: Velocity profiles for different values of Schmidt (Sc) Number

The influence of the Schmidt number Sc on the velocity is plotted in Figure 4.1.7. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. It is worth to mention that for hydrogen ($Sc = 0.22$) the velocity profiles is much higher than that of other Sc. From these figures, it is obvious that the buoyancy forces parameters enhances the fluid velocity thereby increasing the momentum boundary layer thickness.

4.2 Effects of parameter variation on Temperature Profiles

The numerical results for the temperature profiles are shown in Figures (4.2.1 - 4.2.7). It is seen from these figures that the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary conditions.

4.2.1 Effect of variation of Prandtl number on Temperature

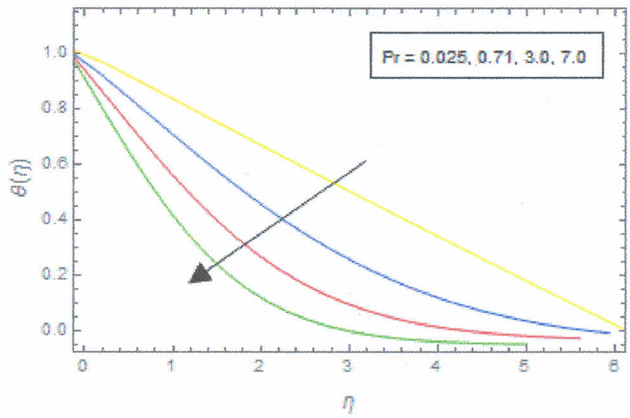

Figure 4.2.1: Temperature profiles for different values of Pr

In figure 4.2.1, we depict the effect of Prandtl number (Pr) on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. This is attributed to the fact that as the Prandtl number decreases, the thermal boundary layer thickness (δ) increases causing reduction in the temperature gradient. The reason is that smaller values of (Pr) are equivalent to increasing the thermal conductivity, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Thus, temperature field falls more rapidly for water in comparison to air and the temperature curve is exactly linear for mercury $(Pr = 0.025)$, which is more sensible towards change in temperature. From this observation it is worthy to conclude that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Air can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. However, air can be better and cheap replacement for industrial purpose. This is because, either increase of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number (Pr). Hence temperature decreases with increasing Prandtl number (Pr).

4.2.2 Effect of variation of angle of Inclination on Temperature

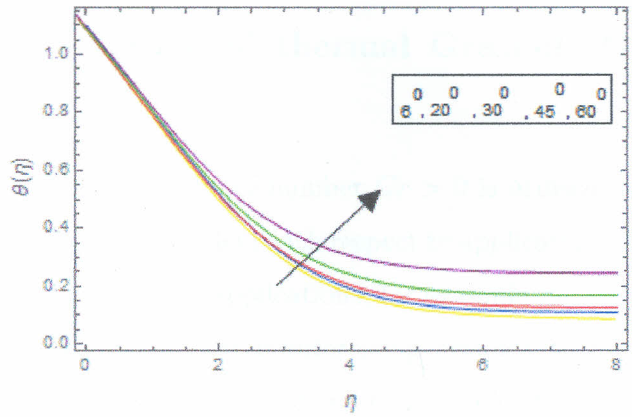

Figure 4.2.2: Temperature profiles for different values γ

It is observed that both the thermal and concentration boundary layer thickness increase as the angle of inclination increases, see Figure 4.2.2.

4.2.3 Effect of variation of Magnetic Parameter on Temperature

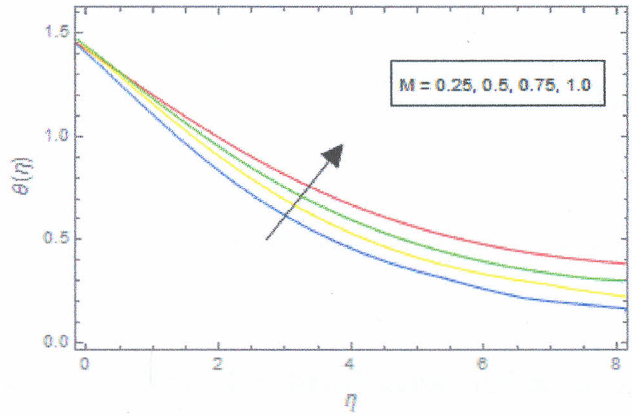

Figure 4.2.3: Temperature profiles for different values of M

From Figure 4.2.3 we see that the temperature increase with the increase of the magnetic field parameter. The presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow and slows down its motion in the boundary layer region. This, in turn, reduces the rate of heat convection in the flow. That is, magnetic field tends to heat the fluid thus, reducing the heat transfer from the wall. This appears as increasing the flow temperature and thermal boundary layer thickness also boosted with increasing M values.

4.2.4 Effect of variation of thermal Grashof (Gr) number on Temperature

The positive values of thermal Grashof number $Gr > 0$ is utilised in our computations. This corresponds to the cooling problem with respect to application. The cooling problem is often encountered in engineering applications; for example in the cooling of electronic components and nuclear reactors. It is interesting however to note that the velocity boundary layer thickness increases (see Figure 4.1.4) while the thermal boundary layer thickness decreases (Figure 4.2.4) with an increase in the value of thermal Grashof number (Gr).

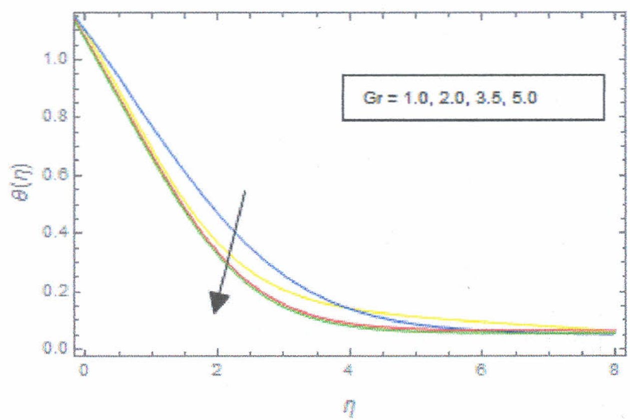

Figure 4.2.4: Temperature profiles for different values of Gr

4.2.5 Effect of variation of solutal Grashof number(Gc) on Temperature

Moreover, an increase in the intensity of buoyancy forces (Gc), causes a decrease in the fluid temperature leading to a decaying thermal boundary layer thickness. The reverse effect is noticed for $Gc > 3.5$.

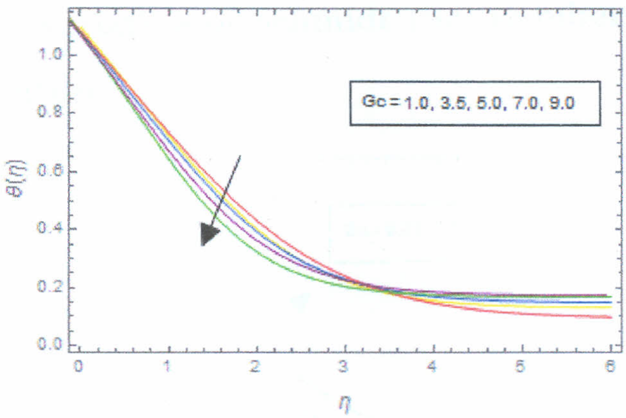

Figure 4.2.5: Temperature profiles for different values of Gc

4.2.6 Effect of variation of Eckert (Ec) number on Temperature

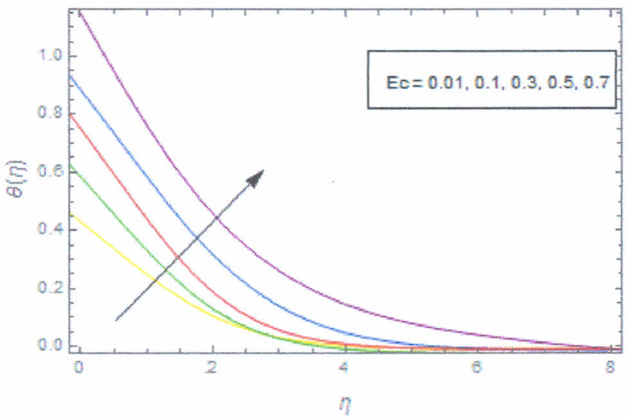

Figure 4.2.6: Temperature profiles for different values of Eckert (Ec)

The effect of Eckert number (Ec) on the temperature is shown in Figure 4.2.6. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature. Therefore, an increase in Eckert number causes an increase in temperature distribution, this is so due to stored heat energy of the fluid that results from frictional heating of fluid particles.

4.2.7 Effect of variation of Schmidt (Sc) number on Temperature

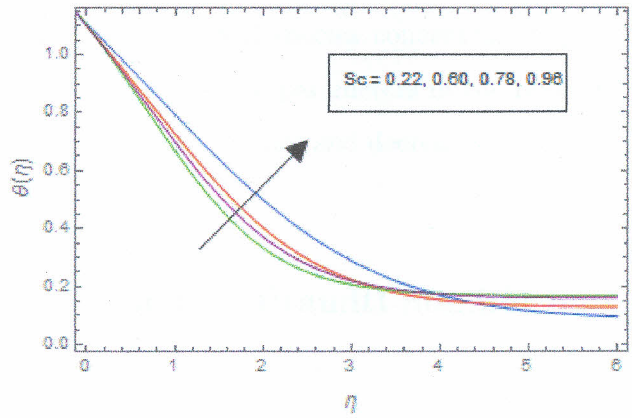

Figure 4.2.7: Temperature profiles for different values of Schmidt (Sc)

Figure 4.2.7 gives the dimensionless temperature profiles for Schmidt number, we observed from these figures that the temperature profile increases with the increase of the Schmidt number. We also observe that the variation in the thermal boundary layer is very small corresponding to a moderate change in Schmidt number. This shows that the minor increasing effect on the temperature profile is greatly affected by the presence of foreign species.

4.3 Effects of parameter variation on Concentration Profiles

Figures (4.3.1 - 4.3.7) depict chemical species concentration profiles against spanwise coordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition.

4.3.1 Effect of variation of Prandtl number on Concentration

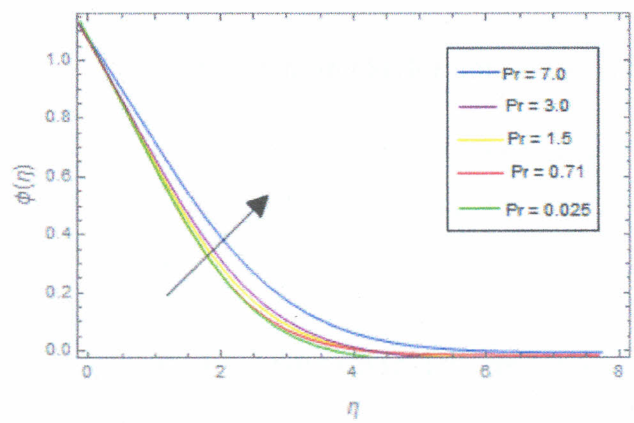

Figure 4.3.1: Concentration profiles for different values of Pr

The increase in the Prandtl number has an adverse effect on the velocity and temperature profiles of the fluid flow but it has opposite effect on the concentration profile of the fluid along the inclined plate as is clear by Figure 4.3.1. In other words, as we increase the Prandtl number, concentration profile has increasing trend. i.e. as Pr increases the thickness of the concentration boundary layer increases.

4.3.2 Effect of variation of angle of Inclination on Concentration

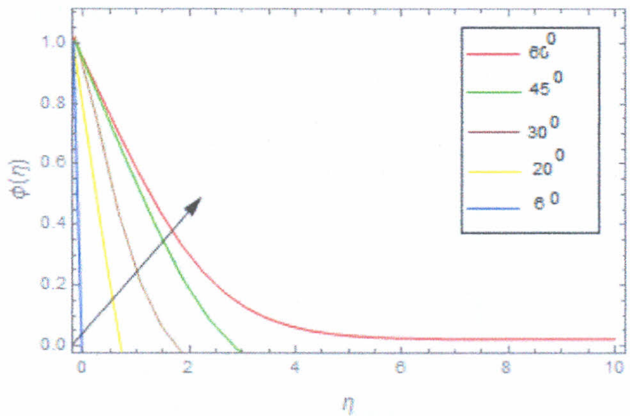

Figure 4.3.2: Concentration profiles for different values γ

An increase in thickness of the concentration boundary layer is observed up on increasing the angle of inclination γ, see Figure 4.3.2. i.e. the concentration of air boundary layer is increased with an increase of γ.

4.3.3 Effect of variation of Magnetic Parameter on Concentration

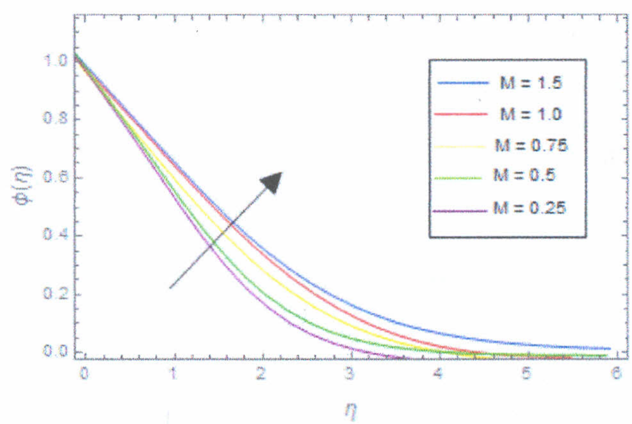

Figure 4.3.3: Concentration profiles for different values of M

However we observed an increase in the concentration boundary layer when the magnetic parameter was increased as graphically displayed in Figure 4.3.3. A Lorenz force produced by the magnetic field retards free convective transfer of fluid mass leaving some molecules stack to the surface of the plate, resulting in the thickening of the concentration layer.

4.3.4 Effect of variation of thermal Grashof (Gr) number on Concentration

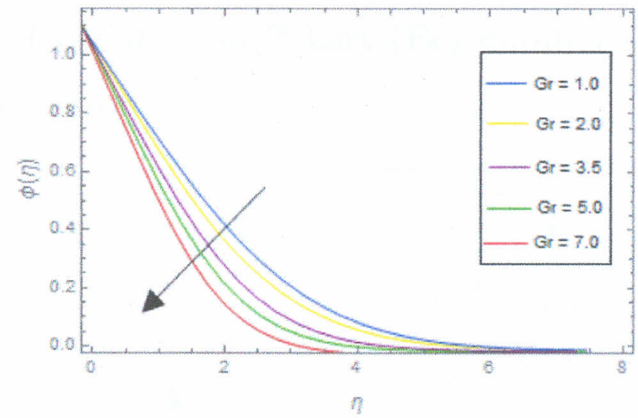

Figure 4.3.4: Concentration profiles for different values of Gr

4.3.5 Effect of variation of solutal Grashof number(Gc) on Concentration

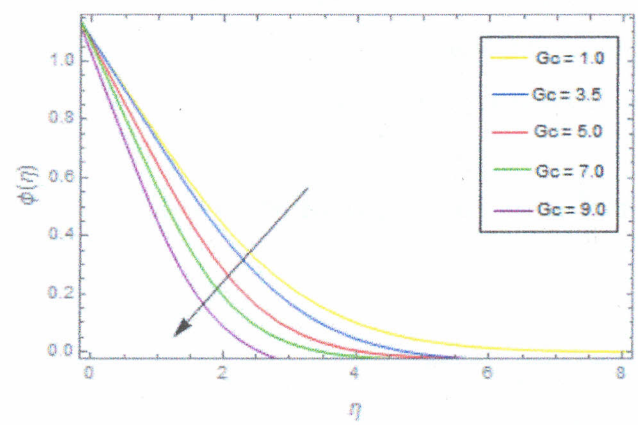

Figure 4.3.5: Concentration profiles for different values of Gc

Attention is focussed on positive values of the buoyancy parameters that is, Grashof number $Gr > 0$ (which corresponds to the cooling problem) and Solutal Grashof number $Gc > 0$ (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface). An increase in the values of thermal and solutal Grashof number (Gr, Gc) due to buoyancy forces also causes a decrease in the chemical species concentration leading to a decaying concentration boundary layer thickness. These observations are displayed in Figures 4.3.4 and 4.3.5.

4.3.6 Effect of variation of Eckert (Ec) number on Concentration

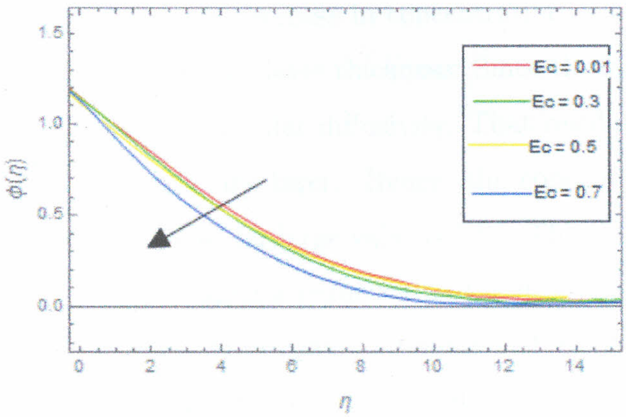

Figure 4.3.6: Concentration profiles for different values of Eckert (Ec)

It is realized that as Eckert number increases, it directly affects the concentration inversely i.e. concentration decreases as a result of increased buoyancy force from enhanced dissipation parameter hence concentration boundary layer reduces, see Figure 4.3.6.

4.3.7 Effect of variation of Schmidt (Sc) number on Concentration

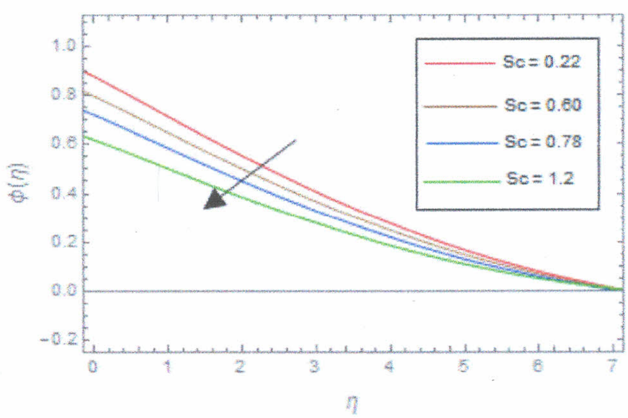

Figure 4.3.7: Concentration profiles for different values of Sc

Figure 4.3.7 concerns with the effect of Sc on the concentration. The Schmidt number therefore quantifies the relative effectiveness of mass transport by diffusion in the concentration (species) boundary layers. It is noted that the concentration at all points in the flow field decreases with the increase of the Schmidt number. It also shows the minor increasing effect on the concentration profile is vastly affected by the presence of foreign species and higher Sc leads to a faster decrease in concentration of the flow field and it reduces the mass concentration boundary layer thickness. Since the increase of Sc means decrease in the chemical species of molecular diffusivity. That results in decrease of the thickness of the concentration boundary layer. Hence, the concentration of species is higher for small values of Sc and lower for large values of Sc. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

In order to bench mark the accuracy of our computed numerical results, the present result has been compared with [8, 5, 32, 2, 58] for different values of parameters and it is observed that the agreements with the solution of velocity, temperature and concentration profiles are excellent.

CHAPTER FIVE

SUMMARY AND RECOMMENDATIONS

5.1 Summary

The convection flows driven by combinations of diffusion effects are very important in many applications. The foregoing formulations may be analyzed to indicate the nature of interaction of the various contributions to buoyancy. In order to gain physical insight into the problem, the value of ϵ was chosen as 1.0. In many practical applications, the characteristics involved, such as the heat transfer rate at the surface are vital since they influence the quality of a final product. The present work, helps us in understanding numerically as well as physically free convection flow in an inclined infinite flat plate in the presence of MHD where viscous dissipation has been employed. The effect of inclination and variation of other controlling physical parameters have been studied and their effects presented. The governing equations for a steady buoyancy driven MHD Heat and Mass Transfer via an inclined flat plate under Boussinesq model were formulated. By suitable similarity scaling transformations, the system of non linear coupled partial differential equations (PDEs) governing the motion of fluid were reduced to a system of coupled non linear ordinary differential equations (ODEs) by reducing the number of independent variables and with appropriate transformed boundary conditions. Furthermore, the similarity equations were solved numerically using shooting method with the fourth order Runge-Kutta numerical method together with the Secant technique of root finding. Numerical evaluations were performed and graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some physical parameters. Based on the results the effects of increasing values of the physical parameters which had significant effect on velocity, temperature and concentration profiles were as follows;

(i) In natural convection flow velocity is sufficiently small, the Prandtl number has no significant effect on concentration. However, it is observed that increase in Prandtl number (Pr) leads to a decrease in velocity and temperature, this is because an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of (Pr) are equivalent to increasing the thermal conductivity, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of (Pr). Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced. This result is in conformity with the known and observed facts that in liquid metals $(Pr < 1)$ the heat diffuses faster as compared to the lubricant oils $(Pr > 1)$.

(ii) We observed that the fluid (air) velocity decreases for an increase in angle of inclination γ. The fluid has higher velocity when the surface is vertical $\gamma = 0$ than when inclined because the convective flow under consideration takes place due to the interaction of gravity and density differences and in the inclined position the effective gravity force is less than what it is when the plate is vertical. On the other hand, both the temperature and concentration profiles increase with an increase of γ. The stability and coherence of the boundary layer depends on the angle of inclination of the surface. When $\gamma < 60^0$, the boundary layer remains stable. The inclination angle $\gamma = 30^0$ gives the enhanced heat and mass distribution of the convective fluid.

(iii) It is to be noted that an increase in the magnetic field has significant effect on the velocity, temperature and concentration profiles. It leads to a rise in temperature and concentration at a slow rate in comparison to the reduction of velocity profiles. In the presence of the magnetic field, the velocity boundary layer is thinner than the temperature and the concentration boundary layer. So magnetic field can effectively be used to control the flow characteristics and heat transfer.

(iv) From the numerical results, it was found that when thermal and solutal Grashof numbers were increased, the thermal and concentration buoyancy effects were enhanced and thus, the fluid velocity increased. Also, when the Schmidt number was increased, the concentration level was decreased resulting in decreased fluid velocity.

(v) It was noted that an increase in Eckert number enhanced the velocity and temperature profiles but a decrease in concentration.

(vi) An increase in Schmidt number results in lowering the concentration and velocity while temperature of the fluid increases. Therefore, Schmidt number has greater effect on concentration profiles than the velocity and temperature profiles. So, we can dominate the rate of mass transfer with the help of the Schmidt number.

5.2 Recommendations

(i) It is therefore recommended that in applying the technique of inclination to enhance cooling of materials in industrial processes, the range of the cooling angle should be considered and the optimal flow is achieved at acute inclination.

(ii) The Schmidt number which enhances mass diffusivity should be considered in processes involving fluid transportation.

(iii) The viscous dissipation parameter had an integral effect in increasing the temperature in the boundary layer and therefore should be considered in the design of heating systems.

(iv) The idea of geometries in fluid flow also provide a test ground for checking the validity of theoretical analyses. Therefore, an effort is needed to explore and understand the convective heat and mass transfer processes between a fluid and submerged objects of various shapes.

(v) An attempt should be made to solve this problem using other numerical techniques as finite difference, finite element, implicit methods among others and compare results.

(vi) Finally, a further research can be undertaken to take care of mixed convection flows with improved methods or unique boundary conditions, then compare results.

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