Jumps in the Kenyan Interest Rates

Apaka Rangita[1], Silas N. Onyango[2], Omolo Ongati[3] and Otula Nyakinda[4]

[1,3,4]School of Mathematics and Actuarial Science, Jaramogi Oginga Odinga University of Science and Technology, P.O. Box 210-40601 Bondo, Kenya

[2]KCA University, P.O. Box 56808 Ruaraka, Nairobi

[1]rangitta@gmail.com, [2]SOnyango@strathmore.edu, [3]nomoloongati@gmail.com, [4]jnyakindas@yahoo.com

Abstract— **In this paper, we use the Bernoulli Jump Diffusion (BJD) process to test for the existence and probability of jumps in the Kenyan interest rates. We test these using the Maximum Likelihood Estimation (MLE) method on the weekly changes in the 91 day Kenyan treasury bills rates. We also compare the BJD process and the Pure Diffusion Process (PDP) in modeling these interest rates. We use the statistical software Eviews 6 to analyze the data.**

***Keywords*— Jumps, Models, Diffusion and Bernoulli**

***Mathematics Subject Classification*: Primary 97K80, Secondary 97M30, 97K60, 91B26, 60G57**

I. INTRODUCTION

The Black–Scholes-Merton stock price model has been used and improved upon by various scholars to model interest rates, exchange rates, options and other financial derivatives. In particular, it has been used to create jump diffusion processes used in the empirical study of interest rates. Jumps refer to the tendency of interest rates, stocks and asset prices to deviate suddenly from a normal path due to non systematic risk such as the arrival of new information. The arrival of such information can be modeled using the Bernoulli or Poisson processes.

Merton [6] assumes that interest rates follow a simple Gaussian random walk with constant drift given by the equation:

$$dr_t = \alpha dt + \sigma dZ_t \qquad (1)$$

Where α is the drift, σ is the volatility and Z_t is a standard one- dimensional Brownian motion i.e. with mean 0 and variance 1.

The arbitrage price $P(t,T)$ of a T-maturity zero-coupon bond in this model can be written as:

$$P(\tau) = \exp\left(-r_t\tau - \frac{1}{2}\alpha\tau^2 + \frac{1}{6}\sigma^2\alpha^3\right)$$

where $\tau = T - t$, this gives the yield of a zero-coupon bond as:

$$Y_\tau = r_t + \frac{1}{2}\alpha\tau - \frac{1}{6}\sigma^2\tau^2$$

which is log-normally distributed. This property helps in the explicit computation of bond prices.

Ball and Torous [1] developed on this by adding jumps and modeled stock price returns using the equation:

$$\frac{dS}{S} = (\alpha)dt + \sigma dZ_t + dN_t \qquad (2)$$

Where α is the instantaneous expected return on the stock, σ^2 is the instantaneous variance of the return conditional on no arrivals of important new information, Z_t is the standard Wiener process, N_t is a Bernoulli process governing the arrival of important new information, hence jumps. Over a fixed period of time (t) at most one jump can occur with probability λt, where λ is the rate of the process. Setting the mean logarithmic jump size to zero, their model results in a daily security return whose density function is a mixture of Gaussian densities;

$$f(x) = (1-\lambda)\phi(x;\alpha,\sigma^2) + \lambda\phi(x;\alpha,\sigma^2+\delta^2)$$

where,

$$\phi(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Beckers [2] had earlier used the same model but with the jumps as a Poisson process.

In the empirical examination of jump diffusion models standard MLE is used to determine the parameters from the discretised data taken at given time intervals. However, Honore [5] asserts that this method is invalid because in jump diffusion models the log-return is equivalent to a discrete mixture of N normally distributed variables where N goes to infinity and the likelihood function is unbounded. He suggests that we need to assume that if the variance of the jump sizes is distributed as $N(\alpha,\sigma^2)$ it can be related to the variance of the diffusion component which is then distributed as $N(\beta, m\sigma^2)$, where m is an arbitrary value which maximizes the log-likelihood profile. He models stock prices using the equation:

$$\frac{dS_t}{S_o} = \alpha dt + \sigma dZ_t + J_t dN_t \quad (3)$$

where α is the drift term, σ is the volatility of the diffusion part, Z_t is the standard Wiener process, $J_t dN_t$ is the jump component and t_0 denotes the nearest point of time preceding t.

It is assumed that $dN_t \sim Poisson\ (\lambda\ dt)$ and that the jump (J_t) amplitude is log-normally distributed as $\log(1 + J_t) \sim N(\mu, \delta^2)$. The stock price therefore has a continuous diffusion part responsible for the usual price fluctuations and the discontinuous jump part caused by extreme events.

The solution to equation (3) is:

$$\log(S_t) = \log(S_0) + \left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma Z_t + \sum \log\left(1 + J_s dN_t\right)$$

Where summation part is in the interval $0 < s \leq t$. Honore, uses a new MLE procedure to estimate the parameters $(\alpha, \sigma, \mu, \delta, \lambda)$. Stock price (S_t) is observed at discrete times $t_i = i\Delta$ for $i = 0,1, \ldots T$ where Δ is the sampling frequency. Letting S_t denote an observation of S at time t_i, the density function for the log-return, $x_i = log\frac{S_i}{S_{i-1}}$ is given as:

$$p(x_i\theta) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\Delta}(\lambda\Delta)^n}{n!} \phi(x_i; (\alpha - \frac{\sigma^2}{2})\Delta + n\mu, \sigma^2\Delta + n\delta^2)$$

Where, $\phi\ (x; m, v^2)$ is the density function for a normally distributed variable with meam m and variance v^2. Honore also used a Bernoulli jump diffusion approximation to this process which is reasonable if $\lambda\Delta \approx 0$. In which case the density function becomes;

$$p(x_i\theta) = (1 - \lambda\Delta)\phi(x_i; (\alpha - \frac{\sigma^2}{2})\Delta, \sigma^2\Delta) + \lambda\Delta\phi(x_i; (\alpha - \frac{\sigma^2}{2})\Delta, \sigma^2\Delta + \delta^2) \quad (4)$$

The log-likelihood function of (4) can be written as:

$$L(x_1, x_2, \ldots x_T;\ \theta) = \sum_{i=1}^{T} \log p(x_i, \theta) \quad (5)$$

Where $\theta = (\alpha, \sigma, \mu, \delta, \lambda)$,

Normal MLE would entail maximizing equation (5) with respect to θ. However, Honore explains that it is invalid to use standard MLE here since the log-likelihood function is unbounded. The remedy is that for a fixed positive m, let $\delta^2 = m\sigma^2$ so that the relative size of the volatilities is fixed. The new log-likelihood function becomes; $L(x_1, x_2 \ldots x_T;\ \theta^*)$, where $\theta^* = (\alpha, \sigma, \mu, \lambda, m\sigma^2)$. The consistent estimator of θ^* is found by choosing m which maximizes this function.

II. THE MODELS

Peter [7] conducts an empirical study of the dynamics of the Australian interest rates of six different bills and bonds, he uses the unique MLE method pioneered by Honore on a variety of models nested in the Chan, Karolyi, Longstaff and Sanders (CKLS) model [3] focusing on the daily, weekly and monthly changes in the rates. He finds very strong evidence of jumps in all the daily interest rates.

Of interest in this study are two models that Peter [7] used which are the PDP and the BJD models.

In the PDP model interest rates are assumed to follow the process:

$$dr_t = \mu_1 + \sigma dZ_t \quad (6)$$

Where, μ_1 is the drift and σ is the volatility.

The discrete form of equation (6) can be written as:

$$x_t = \Delta r_t = r_t - r_{t-\tau} = \mu_1\tau + \epsilon\sigma\sqrt{\tau}$$

Where $\epsilon \sim N(0,1), \Delta t = \tau$, hence $x_t \sim N(\tau\mu_1, \tau\sigma^2)$

$\tau = \frac{1}{252}, \frac{1}{52}, \frac{1}{12}$ respectively if we sample at daily weekly and monthly intervals. The density function of x_t is given by;

$$f(x_t) = \phi(x_t; \tau\mu_1, \tau\sigma^2)$$

Where $\phi(x; \mu, \sigma^2)$ is the density function for $x \sim N(0,1)$ and can be written explicitly as:

$$f(x_t) = \frac{1}{\sigma\sqrt{2\pi\tau}} \exp\left(-\frac{(x_t - \tau\mu_1)^2}{2\tau\sigma^2}\right)$$

Whose log-likelihood function is:

$$LLF = \sum_{i=1}^{n}\left[\frac{1}{\sigma\sqrt{2\pi\tau}} \exp\left(-\frac{(x_t - \tau\mu_1)^2}{2\tau\sigma^2}\right)\right]$$

and the parameters to estimate are;

$\theta = (\mu_1, \sigma)$

If we assume that interest rates follow a BJD process then equation (6) becomes:

$$dr_t = \mu_1 + \sigma dZ_t + JdN_t \quad (7)$$

Where $\mu_1$is the drift, σ is the volatility, $J \sim N(\tau\mu_2, \tau m\sigma^2)$ is the Bernoulli process, μ_2 is the estimated mean of the jump sizes, N is assumed to be independent of the Brownian motion Z_t. The Bernoulli process N_t has the distribution $P(\Delta N_t = 1) = \omega$ and $(\Delta N_t = 0) = 1 - \omega$, so that there can be at most one jump in an interval. Let $\omega = \lambda\tau$, where τ is the length of the interval and ω is the probability that there is a jump in an interval of length τ. The discrete form of equation (7) is:

$$x_t = \Delta r_t = r_t - r_{t-\tau} = \mu_1\tau + \epsilon\sigma\sqrt{\tau} + J\Delta N_t$$

Where $\epsilon \sim N\,(0,1), \Delta t = \tau$

The sum of the two independent normal distributions which are $N(\tau\mu_1, \tau\sigma^2)$ and $N(\tau\mu_2, \tau m\sigma^2)$ will give a normal distribution:

$$N(\tau(\mu_1 + \mu_2), \tau(1+m)\sigma^2)$$

We have that$\Delta N_t = 1$ with probability ω and $\Delta N_t = 0$ with probability $1-\omega$, so that the density function of x_t will be a mixture of two normals;

$$f(x_t) = (1-\omega)\phi(x_t; \tau\mu_1, \tau\sigma^2) + \omega\phi(x_t; \tau(\mu_1+\mu_2), \tau(1+m)\sigma^2)$$

This density function can also be written as;

$$f(x_t) = (1-\omega)\frac{1}{\sigma\sqrt{2\pi\tau}}\exp\left(-\frac{(x_t-\tau\mu_1)^2}{2\tau\sigma^2}\right) + \omega\frac{1}{\sigma\sqrt{2\pi\tau(1+m)}}\exp\left(-\frac{(x_t-\tau(\mu_1+\mu_2))^2}{2\tau(1+m)\sigma^2}\right) \quad (8)$$

Whose log-likelihood function is:

$$LLF = \sum_{i-1}^{n}\log\left\{\log(1-\omega)\frac{1}{\sigma\sqrt{2\pi\tau}}\exp\left(-\frac{(x_i-\tau\mu_1)^2}{2\tau\sigma^2}\right) + \omega\frac{1}{\sigma\sqrt{2\pi\tau(1+m)}}\exp\left(-\frac{(x_i-\tau(\mu_1+\mu_2))^2}{2\tau(1+m)\sigma^2}\right)\right\}$$

In this case the parameters to estimate are $\theta = (\mu_1, \mu_2, \omega, \sigma)$ Das [4] argues that the choice of jump diffusion as the stochastic for interest rates is a natural one, since information flows affect interest rates continuously in small amounts best described by diffusion processes, yet on rare occasions surprise information have large economic impact causing interest rates to jump. These models have been used to conduct empirical studies in several developed countries but rarely in the developing countries. It is for this reason that we opt to test these models in Kenya.

III. EMPIRICAL RESULTS

We used secondary data of weekly interest rates of the 91 day Kenyan treasury bills covering the period from January 1998 to October 2013 from the central bank of Kenya, giving a total of 823 observations. We focus on the weekly changes in these rates as opposed to the rates themselves giving us 822 observations. The weekly changes have a mean of -0.0199 and a standard deviation of 39.36 basis points. While maximum weekly increase in yields is 194 basis points and the maximum weekly drop in yields is 200 basis points, these are large weekly moves which helps explain the high Kurtosis of 6.561. These changes are also negatively skewed (-0.2585)

We used the statistical software Eviews 6 at 5% level of significance to estimate the parameters. We established the profile for the log likelihood function for the PDP and BJD models which we coded in the software. By manual adjustment we found m to be 24 for the BJD model.

From the empirical results we note that the estimates for the drift (μ_1) is 0.1405 for the PDP and -0.3588 for the BJD model. The estimate for volatility (σ) is quite large for the PDP model at1.583. A comparison of the PDP and the BJD reveals a sharp drop of about two-thirds to 0.5337 in volatility when the jump is introduced, suggesting that jumps account for a substantial component of volatility. For the BJD model the standard deviation of the jump component is 2.165 (given by $\sigma\sqrt{m}$ where σ is the standard deviation of the pure diffusion component of the jump diffusion model).

The estimated mean of the jump sizes (μ_2) is -0.4431for the BJD which is quite large. The mean number of abnormal information arrivals per week (λ) is approximately 7 and the estimate for the weekly mean number of jumps ($\frac{\lambda}{52}$) is 0.135 for the BJD. For the same model the probability of a jump ($\omega = \lambda\tau$) in any given week is 13.65 percent which is a strong evidence of the existence of jumps which are also quite large as seen in the μ_2 value.

Finally, based on the Schwarz criterion in which smaller values are preferred the BJD model performs better than the PDP.

IV. CONCLUSION

We have been able to confirm the existence and find probability of jumps in the Kenyan interest rates using the PDP and BJD models with the 91 day treasury bills as the bench mark. Such studies should be done continuously in different countries for bills and bonds since there might be different dynamics acting on the various financial derivatives at different times.

REFERENCES

[1]. Ball C. A., Torous W. N, (1983). A simplified jump process for common stock returns; *Journal of Financial and Quantitative Analysis*, Vol. 18, pp. 53-65.

[2]. Beckers S. (1981). A note on estimating the parameters of the diffusion-jump model of stock returns, *Journal of Financial and Quantitative Analysis*, Vol. 16, pp. 127-140.

[3]. Chan K.C., Karolyi G.A., Longstaff F. A., Sanders A. B., (1992). An empirical comparison of alternative models of the short term interest rate. *Journal of Finance*, Vol. 47, pp. 1209-1227.

[4]. Das S. R. (2000). *Interest rate modeling with jump-diffusion processes*. Wiley, New York.
[5]. Honore P. (1998). Pitfalls in estimating jump-diffusion models, *Working paper*, Center for analytical finance, Aarhus School of Business, Denmark.
[6]. Merton R. C. (1973). Option pricing when underlying stock returns are discontinuous, *Journal of financial Economics*, Vol. 3, pp. 125-144.
[7]. Peter T. O. (2006). Term structure modeling and the dynamics of Australian interest rates. *PhD Thesis*. University of new South Wales, pp. 82-132.