NUMERICAL SOLUTIONS OF BURGERS EQUATION USING HYBRID METHODS

ABSTRACT

Research on Burgers equation: $u_t + uu_x = \alpha u_{xx}$ which is a second order nonlinear partial differential equation is gaining interest for its application and mathematics itself. It arises in model studies of turbulence and shock wave theory. In physical application of shock waves in fluids, the coefficient α has the meaning of viscosity. For light fluids or gases the solution considers the inviscid limit as α tends to zero. The solution of Burgers equation can be classified into two categories: Numerical solutions using both finite difference and finite elements approaches; the analytic solution found by Cole and Hopf. Analytical solutions for the equation have so far been found for the value of $\alpha \in [0, 1]$. Numerically stable solutions were also obtained only for the value of $\alpha \in [0, 1]$. In this thesis, we have obtained solution to the Burgers equation numerically for $\alpha \in [0, 10]$. In pursuit of our objective, we have used hybrid methods resulting from Crank-Nicolson, Du Fort-Frankel and Lax-Friedrich directly to the Burgers equation. This method applies Crank-Nicolson finite difference scheme directly on the nonlinear equation and derive a nonlinear finite difference scheme, and then use it to derive a system of linear equations. The results of the hybrid schemes are found to compare well with the exact solution. Therefore, this thesis is an improvement of the results obtained by other researchers in the application of Burgers equation.