ABSRACT

A nonlinear Black-Scholes Partial Differential Equation whose nonlinearity is as a result of transaction costs and a price slippage impact that lead to market illiquidity with feedback effects was studied. Most of the solutions obtained in option pricing especially using nonlinear equations are numerical which gives approximate option values. To get exact option values, analytic solutions for these equations have to be obtained. Analytic solutions to the nonlinear Black-Scholes Partial Differential Equation for pricing call and put options to expiry time are currently unknown. The main purpose of this study was to obtain analytic solutions of European call and put options of a nonlinear Black-Scholes Partial Differential Equation with transaction costs and a price slippage impact. The methodology involved reduction of the equation into a second-order nonlinear Partial Differential Equation. By assumption of a traveling wave profile the equation was further reduced to Ordinary Differential Equations. Solutions to all the transformed equations gave rise to an analytic solution to the nonlinear Black-Scholes equation for a call option. Using the put-call parity relation the put option's value was obtained. The solutions obtained will be used to price put and call options in the presence of transaction costs and a price slippage impact. The solutions may also help in fitting the Black-Scholes option pricing model in the modern option pricing industry since it incorporates real world factors hence significantly contributing to the field of mathematical finance. We, therefore, recommend to hedgers and speculators in derivatives markets to make use of option pricing formulae obtained in this research for accurate option pricing so that they can maximize their profits. In conclusion, further research needs to be done to study the exposure from writing a covered call and the exposure from writing a naked put.